

Perturbative renormalization factors of four-quark operators for improved Wilson fermion action and Iwasaki gauge action

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(Dated: February 25, 2013)

Abstract

The renormalization factor and $O(a)$ improvement coefficient of four-quark operators are calculated perturbatively for the improved Wilson fermion action with clover term and the Iwasaki gauge action. With an application to the $K \rightarrow \pi\pi$ decay amplitude in mind, the calculation is restricted to the parity odd operator, for which the operators are multiplicatively renormalized without any mixing to operators that have different chiral structures.

I. INTRODUCTION

Calculation of hadron matrix elements of phenomenological interest is one of major applications of lattice QCD. When one tries to calculate weak matrix elements of four quark operators using the Wilson fermion action, however, one encounters an obstacle since unwanted mixings with operators having wrong chirality, which are prohibited in the continuum, is generally introduced through quantum corrections.

This problem is absent for parity odd operators. Using the discrete symmetries of parity, charge conjugation and flavor exchanging transformations, it was shown [1] that the parity odd four quark operator has no extra mixing with wrong chirality operators even without chiral symmetry. This is a welcome feature for calculation of the $K \rightarrow \pi\pi$ decay amplitude with the Wilson fermion action.

An improvement with the clover term is indispensable for the Wilson fermion action. Since the renormalization group improved gauge action of Iwasaki type [2] has good scaling property already at lattice spacings around $a^{-1} \sim 2$ GeV, the combination of the Iwasaki gauge action and the improved Wilson fermion action with clover term is a plausible choice for numerical simulations. Unfortunately renormalization factors of the four quark operators are not available for this combination of the actions. In this paper we calculate the renormalization factor of four quark operators which contribute to the $K \rightarrow \pi\pi$ decay to one-loop order in perturbation theory.

This paper is organized as follows. In Sec. II we briefly introduce the action and the Feynman rules relevant for the present calculation. In Sec. III the $\Delta S = 1$ four quark operators are introduced, which contribute to $K \rightarrow \pi\pi$ decay. The one loop contributions are briefly reviewed in Sec. IV for gluon exchange diagrams and are calculated in Sec. V for penguin diagrams. The renormalization factors are evaluated in Sec. VI for the $\overline{\text{MS}}$ scheme. Sec. VII is devoted for an evaluation of $O(a)$ effect. Our conclusion is in Sec. VIII.

The physical quantities are expressed in lattice units and the lattice spacing a is suppressed unless necessary. We take $\text{SU}(N)$ gauge group with the gauge coupling g and the second Casimir $C_F = \frac{N^2 - 1}{2N}$, while $N = 3$ is specified in the numerical calculations.

II. ACTION AND FEYNMAN RULES

We adopt the Iwasaki gauge action

$$S_{\text{gluon}} = \frac{1}{g^2} \left\{ c_0 \sum_{\text{plaquette}} \text{Tr} U_{pl} + c_1 \sum_{\text{rectangle}} \text{Tr} U_{rtg} \right\} \quad (\text{II.1})$$

with $c_1 = -0.331$ [2] and the improved Wilson fermion action with the clover term

$$S_{\text{fermion}} = \sum_n \bar{\psi}(n) \left(\gamma_\mu D_\mu - \frac{r}{2} D^2 + m_0 \right) \psi(n) - c_{\text{SW}} \sum_n \sum_{\mu, \nu} i g \frac{r}{4} \bar{\psi}_n \sigma_{\mu\nu} P_{\mu\nu}(n) \psi_n, \quad (\text{II.2})$$

where $\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]$. We shall set $r = 1$ in the following.

Weak coupling perturbation theory is developed by expanding the action in terms of gauge coupling. The gluon propagator of the Iwasaki action is given as an inverse of the action kernel

$$G_{\mu\nu}^{AB}(p) = (D^{-1}(p))_{\mu\nu} \delta^{AB}, \quad (\text{II.3})$$

$$S_{\text{gluon}}^{\text{free}} = \frac{1}{2} \int_{-\pi}^{\pi} \frac{d^4 p}{(2\pi)^4} \sum_{\mu, \nu} A_\mu^B(p) D_{\mu\nu}(p) A_\nu^B(-p), \quad (\text{II.4})$$

$$D_{\mu\nu}(p) = \hat{p}_\mu \hat{p}_\nu + \sum_{\rho} (\hat{p}_\rho \delta_{\mu\nu} - \hat{p}_\mu \delta_{\rho\nu}) q_{\mu\rho} \hat{p}_\rho, \quad (\text{II.5})$$

$$q_{\mu\nu} = (1 - \delta_{\mu\nu}) (1 - c_1 (\hat{p}_\mu^2 + \hat{p}_\nu^2)), \quad (\text{II.6})$$

$$\hat{p}_\mu = 2 \sin \frac{p_\mu}{2}, \quad (\text{II.7})$$

where we adopted the Feynman gauge.

The quark propagator is given by that of the ordinary Wilson fermion action

$$S_F(p) = \frac{-i \sum_{\mu} \gamma_{\mu} \bar{p}_{\mu} + W(p)}{\bar{p}^2 + W(p)^2}, \quad (\text{II.8})$$

$$\bar{p}_{\mu} = \sin p_{\mu}, \quad (\text{II.9})$$

$$W(p) = M + r \sum_{\mu} (1 - \cos p_{\mu}). \quad (\text{II.10})$$

We only need single gluon vertexes for our present calculation, one from the standard gauge coupling

$$V_{1\mu}^A(k, p) = -i g T^A \left(\gamma_{\mu} \cos \frac{1}{2} (-k_{\mu} + p_{\mu}) - i r \sin \frac{1}{2} (-k_{\mu} + p_{\mu}) \right) \quad (\text{II.11})$$

and another from the clover term

$$V_{1\mu}^{(c)A}(k, p) = -g T^A c_{\text{SW}} \frac{r}{2} \left(\sum_{\nu} \sigma_{\mu\nu} \sin(p_{\nu} + k_{\nu}) \right) \cos \frac{1}{2} (p_{\mu} + k_{\mu}), \quad (\text{II.12})$$

where k and p represent incoming momentum into the vertex as is shown in Fig. 1. T^A ($A = 1, \dots, N^2 - 1$) is a generator of color $SU(N)$.

III. FOUR-QUARK OPERATORS

We shall evaluate the renormalization factor of the following ten operators

$$Q^{(1)} = (\bar{s}d)_L (\bar{u}u)_L, \quad Q^{(2)} = (\bar{s} \times d)_L (\bar{u} \times u)_L, \quad (\text{III.1})$$

$$Q^{(3)} = (\bar{s}d)_L (\bar{u}u + \bar{d}d + \bar{s}s)_L, \quad Q^{(4)} = (\bar{s} \times d)_L (\bar{u} \times u + \bar{d} \times d + \bar{s} \times s)_L, \quad (\text{III.2})$$

$$Q^{(5)} = (\bar{s}d)_L (\bar{u}u + \bar{d}d + \bar{s}s)_R, \quad Q^{(6)} = (\bar{s} \times d)_L (\bar{u} \times u + \bar{d} \times d + \bar{s} \times s)_R, \quad (\text{III.3})$$

$$Q^{(7)} = (\bar{s}d)_L \left(\bar{u}u - \frac{1}{2}\bar{d}d - \frac{1}{2}\bar{s}s \right)_R, \quad Q^{(8)} = (\bar{s} \times d)_L \left(\bar{u} \times u - \frac{1}{2}\bar{d} \times d - \frac{1}{2}\bar{s} \times s \right)_R, \quad (\text{III.4})$$

$$Q^{(9)} = (\bar{s}d)_L \left(\bar{u}u - \frac{1}{2}\bar{d}d - \frac{1}{2}\bar{s}s \right)_L, \quad Q^{(10)} = (\bar{s} \times d)_L \left(\bar{u} \times u - \frac{1}{2}\bar{d} \times d - \frac{1}{2}\bar{s} \times s \right)_L, \quad (\text{III.5})$$

where

$$(\bar{s}d)_{R/L} = \bar{s}\gamma_\mu (1 \pm \gamma_5) d \quad (\text{III.6})$$

and \times means the following contraction of the color indices

$$Q^{(2)} = (\bar{s} \times d)_L (\bar{u} \times u)_L = (\bar{s}_a d_b)_L (\bar{u}_b u_a)_L. \quad (\text{III.7})$$

We notice that these operators are not all independent, satisfying the relations

$$Q^{(4)} = -Q^{(1)} + Q^{(2)} + Q^{(3)}, \quad (\text{III.8})$$

$$Q^{(9)} = \frac{1}{2} (3Q^{(1)} - Q^{(3)}), \quad (\text{III.9})$$

$$Q^{(10)} = \frac{1}{2} (3Q^{(2)} - Q^{(4)}). \quad (\text{III.10})$$

We are interested in the parity odd operators, which contribute to the $K \rightarrow \pi\pi$ decay amplitude

$$Q_{VA+AV}^{(2n-1)} = -Q_{VA}^{(2n-1)} - Q_{AV}^{(2n-1)}, \quad Q_{VA+AV}^{(2n)} = -Q_{VA}^{(2n)} - Q_{AV}^{(2n)}, \quad (n = 1, 2, 5) \quad (\text{III.11})$$

$$Q_{VA-AV}^{(2n-1)} = Q_{VA}^{(2n-1)} - Q_{AV}^{(2n-1)}, \quad Q_{VA-AV}^{(2n)} = Q_{VA}^{(2n)} - Q_{AV}^{(2n)}, \quad (n = 3, 4), \quad (\text{III.12})$$

$$Q_{VA}^{(2n-1)} = (\bar{s}d)_V \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q}q)_A, \quad (\text{III.13})$$

$$Q_{AV}^{(2n-1)} = (\bar{s}d)_A \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q}q)_V, \quad (\text{III.14})$$

$$Q_{VA}^{(2n)} = (\bar{s} \times d)_V \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q} \times q)_A, \quad (\text{III.15})$$

$$Q_{AV}^{(2n)} = (\bar{s} \times d)_A \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q} \times q)_V, \quad (\text{III.16})$$

where the coefficients are given for $q = u, d, s$

$$\alpha_q^{(1)} = (1, 0, 0), \quad (\text{III.17})$$

$$\alpha_q^{(2)} = \alpha_q^{(3)} = (1, 1, 1), \quad (\text{III.18})$$

$$\alpha_q^{(4)} = \alpha_q^{(5)} = \left(1, -\frac{1}{2}, -\frac{1}{2}\right) \quad (\text{III.19})$$

and current-current vertex means

$$(\bar{s}d)_V (\bar{q}q)_A = (\bar{s}\gamma_\mu d) (\bar{q}\gamma_\mu \gamma_5 q). \quad (\text{III.20})$$

There are two kinds of one loop corrections to these operators. One is given by gluon exchanging diagrams in Fig. 2 and the other by the penguin diagrams in Fig. 3. The gluon exchanging diagrams preserve the flavor structure and there occur no mixing between operators with different n . On the other hand the penguin diagrams mix any operator to the penguin operator unless it belongs to a different representation of the flavor $SU(3)_f$.

There are a number of parity odd operators having different chirality

$$O_{SP\pm PS}^{(2n-1)} = (\bar{s}d)_S \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q}q)_P \pm (\bar{s}d)_P \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q}q)_S, \quad (\text{III.21})$$

$$O_{SP\pm PS}^{(2n)} = (\bar{s} \times d)_S \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q} \times q)_P \pm (\bar{s} \times d)_P \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q} \times q)_S, \quad (\text{III.22})$$

$$O_{T\tilde{T}}^{(2n-1)} = (\bar{s}d)_T \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q}q)_{\tilde{T}}, \quad (\text{III.23})$$

$$O_{T\tilde{T}}^{(2n)} = (\bar{s} \times d)_T \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{q} \times q)_{\tilde{T}}, \quad (\text{III.24})$$

$$(\bar{s}d)_S (\bar{q}q)_P = (\bar{s}d) (\bar{q}\gamma_5 q), \quad (\bar{s}d)_T (\bar{q}q)_{\tilde{T}} = (\bar{s}\sigma_{\mu\nu} d) (\bar{q}\sigma_{\mu\nu} \gamma_5 q) \quad (\text{III.25})$$

Chiral symmetry does not prohibit mixings of these operators in the Wilson fermion system. However, it was shown in Ref. [1] that the Wilson fermion system has sufficient set of discrete symmetries to protect the parity odd operators $Q_{VA\pm AV}$ from such mixings.

For the gluon exchanging diagrams, the operator mixing can be studied with the following operator having four flavors

$$O_{\Gamma'\pm\Gamma\Gamma}^{(o)} = (\bar{\psi}_1\psi_2)_\Gamma (\bar{\psi}_3\psi_4)_{\Gamma'} \pm (\bar{\psi}_1\psi_2)_{\Gamma'} (\bar{\psi}_3\psi_4)_\Gamma, \quad (\text{III.26})$$

$$O_{\Gamma'\pm\Gamma\Gamma}^{(e)} = (\bar{\psi}_1 \times \psi_2)_\Gamma (\bar{\psi}_3 \times \psi_4)_{\Gamma'} \pm (\bar{\psi}_1 \times \psi_2)_{\Gamma'} (\bar{\psi}_3 \times \psi_4)_\Gamma. \quad (\text{III.27})$$

It was proved in Ref. [1] that parity, charge conjugation and two types of flavor exchanging transformations

$$\mathcal{S}' = (\psi_1 \leftrightarrow \psi_2, \psi_3 \leftrightarrow \psi_4), \quad \mathcal{S}'' = (\psi_1 \leftrightarrow \psi_4, \psi_2 \leftrightarrow \psi_3) \quad (\text{III.28})$$

are sufficient to show that the mixings occur only between $O_{VA+AV}^{(o)}$ and $O_{VA+AV}^{(e)}$, or between $O_{VA-AV}^{(o)}$ and $O_{VA-AV}^{(e)}$. We notice that the Fierz rearrangement leads to relations between the Fierz partners

$$O_{VA-AV}^{(e)} = 2O_{SP-PS}^{(o)F}, \quad O_{VA-AV}^{(o)} = 2O_{SP-PS}^{(e)F}, \quad (\text{III.29})$$

$$O_{SP-PS}^{(o)F} = (\bar{\psi}_1\psi_4)_S (\bar{\psi}_3\psi_2)_P - (\bar{\psi}_1\psi_4)_P (\bar{\psi}_3\psi_2)_S. \quad (\text{III.30})$$

Thus O_{VA-AV} and O_{SP-PS} are basically the same operator rearranged with each other, and if we include the Fierz partner O_{VA-AV}^F it also mixes with O_{SP-PS} .

For the penguin diagram we need to keep the three flavors structure. Hence flavor exchange of

$$\mathcal{S}' = (d \leftrightarrow s, \bar{d} \leftrightarrow \bar{s}) \quad (\text{III.31})$$

is a symmetry. Together with parity and charge conjugation one can show that the mixing occurs only among $Q_{VA\pm AV}$'s.

We note that these discrete symmetries still allow mixings with lower dimensional parity odd operators

$$(m_d - m_s) (\bar{s}\gamma_5 d), \quad (\text{III.32})$$

$$(m_d - m_s) \partial_\mu (\bar{s}\gamma_\mu \gamma_5 d), \quad (\text{III.33})$$

$$(m_d - m_s) (\bar{s}F_{\mu\nu}\sigma_{\mu\nu}\gamma_5 d), \quad (\text{III.34})$$

$$(m_d - m_s) \left(\bar{s}\tilde{F}_{\mu\nu}\sigma_{\mu\nu}d \right) \quad (\text{III.35})$$

proportional to a mass difference $(m_d - m_s)$.

IV. ONE LOOP CORRECTION FROM GLUON EXCHANGING DIAGRAMS

We consider the following four fermi operator

$$O_{XY}^{(k)} = (T^{(k)})_{ab;cd} (\Gamma_X \otimes \Gamma_Y)_{\alpha\beta;\gamma\delta} (\bar{s}_{a,\alpha} d_{b,\beta}) (\bar{q}_{c,\gamma} q'_{d,\delta}), \quad (\text{IV.1})$$

in order to evaluate the one loop correction from the gluon exchanging diagrams given in Fig. 2, where T represents the color factor

$$(T^{(2n-1)})_{ab;cd} = (1 \tilde{\otimes} 1)_{ab;cd} = \delta_{ab} \delta_{cd} \quad (\text{for } Q^{(2n-1)}), \quad (\text{IV.2})$$

$$(T^{(2n)})_{ab;cd} = (1 \tilde{\odot} 1)_{ab;cd} = \delta_{ad} \delta_{bc} \quad (\text{for } Q^{(2n)}). \quad (\text{IV.3})$$

and Γ is the gamma matrix

$$(\Gamma_X \otimes \Gamma_Y)_{\alpha\beta;\gamma\delta} = (\Gamma_X)_{\alpha\beta} (\Gamma_Y)_{\gamma\delta}, \quad (\text{IV.4})$$

$$\Gamma_V = \gamma_\mu, \quad \Gamma_A = \gamma_\mu \gamma_5 \quad (\text{IV.5})$$

where summation over μ is taken.

Since the one loop correction has already been evaluated in Ref. [3] for various gauge actions, we just briefly review the result for the Iwasaki gauge action. We consider one loop corrections to the amputated four quark vertex

$$I_{k;XY} = \left\langle O_{XY}^{(k)} s_{a\alpha} \bar{d}_{b\beta} q_{c\gamma} \bar{q}'_{d\delta} \right\rangle_{\text{1PI}}. \quad (\text{IV.6})$$

The contributions from the diagrams (a) and (a') are given by

$$I_{k;XY}^{(a)} = J_k^{(a)} \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} V_{1\mu}(0, l) S_F(l) \Gamma_X S_F(l) V_{1\nu}(-l, 0) \otimes \Gamma_Y G_{\mu\nu}(l), \quad (\text{IV.7})$$

$$I_{k;XY}^{(a')} = J_k^{(a)} \Gamma_X \otimes \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} V_{1\mu}(0, l) S_F(l) \Gamma_Y S_F(l) V_{1\nu}(-l, 0) G_{\mu\nu}(l). \quad (\text{IV.8})$$

The contributions from the diagrams (b) and (b') are

$$I_{k;XY}^{(b)} = J_k^{(b)} \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} V_{1\mu}(0, l) S_F(l) \Gamma_X \otimes \Gamma_Y S_F(l) V_{1\nu}(-l, 0) G_{\mu\nu}(l), \quad (\text{IV.9})$$

$$I_{k;XY}^{(b')} = J_k^{(b)} \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \Gamma_X S_F(l) V_{1\mu}(-l, 0) \otimes V_{1\nu}(0, l) S_F(l) \Gamma_Y G_{\mu\nu}(l). \quad (\text{IV.10})$$

The contributions from the diagrams (c) and (c') are

$$I_{k;XY}^{(c)} = J_k^{(c)} \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} V_{1\mu}(0, l) S_F(l) \Gamma_X \otimes V_{1\nu}(0, -l) S_F(-l) \Gamma_Y G_{\mu\nu}(l), \quad (\text{IV.11})$$

$$I_{k;XY}^{(c')} = J_k^{(c)} \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \Gamma_X S_F(l) V_{1\mu}(-l, 0) \otimes \Gamma_Y S_F(-l) V_{1\nu}(l, 0) G_{\mu\nu}(l). \quad (\text{IV.12})$$

The color index contributions are already factored out in the above

$$J_{2n-1}^{(a)} = T^A 1 T^B \delta^{AB} \tilde{\otimes} 1 = C_F 1 \tilde{\otimes} 1, \quad (\text{IV.13})$$

$$J_{2n}^{(a)} = T^A 1 \tilde{\odot} 1 T^B \delta^{AB} = \frac{1}{2} 1 \tilde{\otimes} 1 - \frac{1}{2N} 1 \tilde{\odot} 1, \quad (\text{IV.14})$$

$$J_{2n-1}^{(b)} = T^A 1 \tilde{\otimes} 1 T^A = \frac{1}{2} 1 \tilde{\odot} 1 - \frac{1}{2N} 1 \tilde{\otimes} 1, \quad (\text{IV.15})$$

$$J_{2n}^{(b)} = T^A 1 T^A \tilde{\odot} 1 = C_F 1 \tilde{\odot} 1, \quad (\text{IV.16})$$

$$J_{2n-1}^{(c)} = T^A 1 \tilde{\otimes} T^A 1 = \frac{1}{2} 1 \tilde{\odot} 1 - \frac{1}{2N} 1 \tilde{\otimes} 1, \quad (\text{IV.17})$$

$$J_{2n}^{(c)} = T^A 1 \tilde{\odot} T^A 1 = \frac{1}{2} 1 \tilde{\otimes} 1 - \frac{1}{2N} 1 \tilde{\odot} 1. \quad (\text{IV.18})$$

Note that contributions should also be included where one or more of the gluon vertexes is replaced with $V_{1\mu}^{(c)}$ from the clover term. We shall adopt an on-shell massless scheme in this section; quark mass and all external momenta are set to zero.

A. Contribution from diagram (a) and (a')

The one loop corrections $I_{XY}^{(a)}$ and $I_{XY}^{(a')}$ are the same as those to the bilinear (axial) vector current operator. Omitting the color factor the one loop contribution is given by

$$I_{VA}^{(a)} = T_V (V \otimes A), \quad I_{VA}^{(a')} = T_A (V \otimes A), \quad (\text{IV.19})$$

$$I_{AV}^{(a)} = T_A (A \otimes V), \quad I_{AV}^{(a')} = T_V (A \otimes V), \quad (\text{IV.20})$$

where $V \otimes A = \gamma_\mu \otimes \gamma_\mu \gamma_5$ and $T_{V/A}$ is the one loop correction to the local (axial) vector current for the Wilson fermion

$$T_\Gamma \Gamma = \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} V_{1\mu}(0, l) S_F(l) \Gamma S_F(l) V_{1\nu}(-l, 0) G_{\mu\nu}(l). \quad (\text{IV.21})$$

Introducing the gluon mass λ to the propagator $G_{\mu\nu}(l)$ in the loop we obtain [4]

$$T_\Gamma = \frac{g^2}{16\pi^2} \left(-\frac{h_2(\Gamma)}{4} \ln(\lambda a)^2 + V_\Gamma \right) \quad (\text{IV.22})$$

in the Feynman gauge, where $h_2(\Gamma)$ is an integer given by

$$h_2(\Gamma) = 4(A), 4(V), 16(P), 16(S), 0(T) \quad (\text{IV.23})$$

for various Dirac channels. The finite constants V_Γ depend quadratically on the clover coefficients c_{SW} , and we write

$$V_\Gamma = V_\Gamma^{(0)} + c_{\text{SW}} V_\Gamma^{(1)} + c_{\text{SW}}^2 V_\Gamma^{(2)}. \quad (\text{IV.24})$$

The superscript ($i = 0, 1, 2$) means a correction of i -th order in c_{SW} , where i gauge interactions are replaced with that from the clover term. The numerical value of the finite part V_{Γ} has already been evaluated in Ref. [4] for various gauge actions and is given in Table I for Iwasaki gauge action.

B. Contribution from diagram (b) and (b')

The contributions from the diagrams (b) and (b') can be evaluated by using the Fierz rearrangement [5] in the spinor indices.

Each one loop correction turned out to be the same as that to the (pseudo) scalar density and (axial) vector current operators given by (IV.21). After carrying out the loop integral we obtain

$$I_{VA+AV}^{(b)} + I_{VA+AV}^{(b')} = -(T_V + T_A) (V \odot A + A \odot V), \quad (\text{IV.25})$$

$$I_{VA-AV}^{(b)} + I_{VA-AV}^{(b')} = -2(T_S + T_P) (S \odot P - P \odot S), \quad (\text{IV.26})$$

where the direct product \odot means

$$(\Gamma \odot \Gamma')_{\alpha\beta;\gamma\delta} = (\Gamma)_{\alpha\delta} (\Gamma')_{\gamma\beta} \quad (\text{IV.27})$$

and $S \odot P = 1 \odot \gamma_5$. Performing the Fierz rearrangement again the vertex function is transformed into the same spinor structure as at the tree level

$$I_{VA+AV}^{(b)} + I_{VA+AV}^{(b')} = (T_V + T_A) (V \otimes A + A \otimes V), \quad (\text{IV.28})$$

$$I_{VA-AV}^{(b)} + I_{VA-AV}^{(b')} = (T_S + T_P) (V \otimes A - A \otimes V). \quad (\text{IV.29})$$

C. Contribution from diagram (c) and (c')

Evaluation of the contributions from the diagrams (c), (c') is performed by using charge conjugation and Fierz rearrangement [5]. We use the representation of the charge conjugation matrix $C = \gamma_2 \gamma_0$ and the relations

$$CS_F(p)C^{-1} = S_F(-p)^T, \quad (\text{IV.30})$$

$$CV_{1\mu}(k, p)C^{-1} = -V_{1\mu}(p, k)^T, \quad (\text{IV.31})$$

$$CV_{1\mu}^{(c)}(k, p)C^{-1} = -V_{1\mu}^{(c)}(p, k)^T. \quad (\text{IV.32})$$

Using these relations and the Fierz rearrangement each quantum correction becomes the same as that to the (pseudo) scalar density and (axial) vector current operators

$$I_{VA+AV}^{(c)} = 2T_S (SC^{-1} \otimes CP) - 2T_P (PC^{-1} \otimes CS) \quad (\text{IV.33})$$

$$I_{VA-AV}^{(c)} = T_V (VC^{-1} \otimes CA) + T_A (AC^{-1} \otimes CV), \quad (\text{IV.34})$$

$$I_{VA+AV}^{(c')} = 2T_P (SC^{-1} \otimes CP) - 2T_S (PC^{-1} \otimes CS) \quad (\text{IV.35})$$

$$I_{VA-AV}^{(c')} = T_A (VC^{-1} \otimes CA) + T_V (AC^{-1} \otimes CV). \quad (\text{IV.36})$$

where the direct product \otimes means

$$(\Gamma \otimes \Gamma')_{\alpha\beta;\gamma\delta} = (\Gamma)_{\alpha\gamma} (\Gamma')_{\delta\beta} \quad (\text{IV.37})$$

Performing the Fierz rearrangement and the charge conjugation the vertex function is transformed into the same spinor structure as at the tree level without any mixing

$$I_{VA+AV}^{(c)} + I_{VA+AV}^{(c')} = -(T_S + T_P) (V \otimes A + A \otimes V), \quad (\text{IV.38})$$

$$I_{VA-AV}^{(c)} + I_{VA-AV}^{(c')} = -(T_V + T_A) (V \otimes A - A \otimes V). \quad (\text{IV.39})$$

V. CONTRIBUTION FROM PENGUIN DIAGRAMS

A. One loop correction

In order to evaluate contributions from the penguin diagram we consider a four-quark operator of the following form

$$Q_{XY}^{(k)} = (T^{(k)})_{ab;cd} (\Gamma_X \otimes \Gamma_Y)_{\alpha\beta;\gamma\delta} \sum_{q=u,d,s} \alpha_q^{(n)} (\bar{s}_{a\alpha} d_{b\beta}) (\bar{q}_{c\gamma} q_{d\delta}), \quad (\text{V.1})$$

where $k = 2n - 1$ or $2n$ and coefficients $\alpha_q^{(n)}$ and $T^{(k)}$ are defined in (III.19) and (IV.3). Then we evaluate the penguin diagram contribution to the amputated four quark vertex function

$$I_{k;XY} = \left\langle Q_{XY}^{(k)} s_{a\alpha}(p_1) \bar{d}_{b\beta}(p_2) q_{c\gamma}(p_3) \bar{q}_{d\delta}(p_4) \right\rangle_{\text{1PI}}, \quad (\text{V.2})$$

which is given by the Feynman diagrams in Fig. 3. All the external momentum are set to in-coming direction and the internal gluon momentum is given by $p = p_1 + p_2 = -(p_3 + p_4)$.

The vertex correction is given in the form

$$I_{2n-1;XY} = J_{\text{pen}} \left(\alpha_d^{(n)} I_{XY;\mu}^{P(2)}(p, m_d) \otimes V_{1\nu}(p_3, p_4) + \alpha_s^{(n)} I_{YX;\mu}^{P(2)}(p, m_s) \otimes V_{1\nu}(p_3, p_4) \right) G_{\mu\nu}(p), \quad (\text{V.3})$$

$$I_{2n;XY} = J_{\text{pen}} \sum_{q=u,d,s} \alpha_q^{(n)} I_{Y;\mu}^{P(1)}(p, m_q) (\Gamma_X \otimes V_{1\nu}(p_3, p_4)) G_{\mu\nu}(p), \quad (\text{V.4})$$

where the color factor is given by

$$J_{\text{pen}} = \left(-\frac{1}{2N} 1 \tilde{\otimes} 1 + \frac{1}{2} 1 \tilde{\odot} 1 \right) \quad (\text{V.5})$$

and we define the following loop integrals

$$I_{Y;\mu}^{P(1)}(p, m_q) = - \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \text{tr} (\Gamma_Y S_q(l-p) V_{1\mu}(-l+p, l) S_q(l)), \quad (\text{V.6})$$

$$I_{XY;\mu}^{P(2)}(p, m_q) = \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} (\Gamma_X S_q(l-p) V_{1\mu}(-l+p, l) S_q(l) \Gamma_Y). \quad (\text{V.7})$$

We notice $I_{1;XY} = 0$. The contributions of $I_{Y;\mu}^{P(1c)}$ and $I_{XY;\mu}^{P(2c)}$ should also be included where the gluon vertex in the loop is replaced with $V_{1\mu}^{(c)}$ from the clover term.

We calculate the following loop integrals

$$I_{V\nu;\mu}^{P(1)}(p, m_q) = -\text{tr} (\gamma_\nu I_\mu^P(p, m_q)), \quad I_{A\nu;\mu}^{P(1)}(p, m_q) = -\text{tr} (\gamma_\nu \gamma_5 I_\mu^P(p, m_q)), \quad (\text{V.8})$$

$$I_{VA;\mu}^{P(2)}(p, m_q) = \gamma_\nu I_\mu^P(p, m_q) \gamma_\nu \gamma_5, \quad I_{AV;\mu}^{P(2)}(p, m_q) = \gamma_\nu \gamma_5 I_\mu^P(p, m_q) \gamma_\nu, \quad (\text{V.9})$$

$$I_\mu^P(p, m_q) = \int \frac{d^4 l}{(2\pi)^4} S_F(l-p) V_{1\mu}(-l+p, l) S_F(l) \quad (\text{V.10})$$

according to the standard procedure of lattice perturbation theory [6], *i.e.*, we expand the functions in terms of the gluon momentum p_μ and the quark mass m .

The vertex functions satisfy the vector Ward-Takahashi identity [6]

$$\sum_{\mu} 2 \sin \frac{p_\mu}{2} I_\mu^P(p, m) = 0, \quad (\text{V.11})$$

$$\sum_{\mu} 2 \sin \frac{p_\mu}{2} I_\mu^{P(c)}(p, m) = 0 \quad (\text{V.12})$$

arising from the identity

$$\sum_{\mu} 2 \sin \frac{p_\mu}{2} V_{1\mu}(-l+p, l) = S_F^{-1}(l-p) - S_F^{-1}(l), \quad (\text{V.13})$$

$$\sum_{\mu} 2 \sin \frac{p_\mu}{2} V_{1\mu}^{(c)}(-l+p, l) = 0. \quad (\text{V.14})$$

Hence the loop corrections should be proportional to

$$\lim_{a \rightarrow 0} I_\mu^P(p, m) \propto (p^2 \delta_{\mu\nu} - p_\mu p_\nu). \quad (\text{V.15})$$

We notice that a term proportional to $\sigma_{\mu\nu} p_\nu$ is also allowed by the identity. However this term gives the same form of contribution as (V.15) when expanded in terms of the external momentum and substituted into (V.8) and (V.9). A detailed discussion shall be given later in Sec. VII B.

The only non-vanishing candidate is

$$I_{Y/XY;\mu}^{P(i)}(p, m) = \frac{1}{2} p_\alpha p_\beta \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} I_{Y/XY;\mu}^{P(i)}(p, m) \Big|_{p=m=0} + \mathcal{O}(a), \quad (\text{V.16})$$

which has logarithmic divergence and should be regularized with some infra-red regulator. We shall adopt the gluon momentum p_μ as a regulator and the regularization term is defined by the similar loop integral

$$I_{Y/XY;\mu}^{P(i)}(p)_{\text{IR}} = \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \theta(\pi^2 - l^2) L_{Y/XY;\mu}^{P(i)}(l, p)_{\text{IR}}, \quad (\text{V.17})$$

where $L_{Y/XY;\mu}^{P(i)}(l, p)_{\text{IR}}$ is the same integrand given in (V.8), (V.9) but with all the Feynman rules replaced with that in the continuum and the quark mass set to zero.

The loop integral is evaluated with a subtraction

$$\begin{aligned} I_{Y/XY;\mu}^{P(i)}(p, m) &= I_{Y/XY;\mu}^{P(i)}(p, m) - I_{Y/XY;\mu}^{P(i)}(p)_{\text{IR}} + I_{Y/XY;\mu}^{P(i)}(p)_{\text{IR}}. \\ &= \frac{1}{2} p_\alpha p_\beta \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} \left(I_{Y/XY;\mu}^{P(i)}(p, m) - I_{Y/XY;\mu}^{P(i)}(p)_{\text{IR}} \right) \Big|_{p=m=0} + \mathcal{O}(a) \\ &\quad + I_{Y/XY;\mu}^{P(i)}(p)_{\text{IR}}. \end{aligned} \quad (\text{V.18})$$

The first term is finite and can be evaluated numerically.

$$\begin{aligned} \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} \left(I_{V_\nu,\mu}^{P(1)}(0) - I_{V_\nu,\mu}^{P(1)}(0)_{\text{IR}} \right) &= \frac{i}{16\pi^2} \left((5.09290(43)) \delta_{\mu\nu} \delta_{\alpha\beta} \right. \\ &\quad \left. - (1.88003(27)) (\delta_{\mu\alpha} \delta_{\nu\beta} + \delta_{\nu\alpha} \delta_{\mu\beta}) \right), \end{aligned} \quad (\text{V.19})$$

$$\frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} \left(I_{A_\nu,\mu}^{P(1)}(0) - I_{A_\nu,\mu}^{P(1)}(0)_{\text{IR}} \right) = 0, \quad (\text{V.20})$$

$$\begin{aligned} \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} \left(I_{VA;\mu}^{P(2)}(0) - I_{VA;\mu}^{P(2)}(0)_{\text{IR}} \right) &= \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} \left(I_{AV;\mu}^{P(2)}(0) - I_{AV;\mu}^{P(2)}(0)_{\text{IR}} \right) \\ &= \frac{1}{2} \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} \left(I_{V_\nu,\mu}^{P(1)}(0) - I_{V_\nu,\mu}^{P(1)}(0)_{\text{IR}} \right) \gamma_\nu \gamma_5. \end{aligned} \quad (\text{V.21})$$

The second term has a logarithmic divergence and is calculated analytically

$$I_{V\nu,\mu}^{P(1)}(p)_{\text{IR}} = \frac{4i}{16\pi^2} \left[(-p^2\delta_{\mu\nu} + p_\mu p_\nu) \frac{1}{3} \left(\ln \frac{\pi^2}{a^2 p^2} + \frac{5}{6} \right) - \frac{p^2}{6}\delta_{\mu\nu} + \frac{1}{2a^2}\delta_{\mu\nu} \right], \quad (\text{V.22})$$

$$I_{A\nu,\mu}^{P(1)}(p)_{\text{IR}} = 0, \quad (\text{V.23})$$

$$I_{VA;\mu}^{P(2)}(p)_{\text{IR}} = I_{AV;\mu}^{P(2)}(p)_{\text{IR}} = \frac{1}{2} I_{V\nu,\mu}^{P(1)}(p)_{\text{IR}} \gamma_\nu \gamma_5. \quad (\text{V.24})$$

We shall drop the last $1/a^2$ divergent term since the corresponding term is absent in the finite part since it is evaluated in terms of a derivative with external momentum.

Contribution from the clover term is given by replacing the gluon interaction vertex with $V_{1\mu}^{(c)}$ in (V.6) and (V.7). The loop integral has no IR divergence and can be evaluated numerically.

$$\frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} I_{V\nu,\mu}^{P(1c)}(0) = \frac{i}{16\pi^2} \left(-(2.90088(27)) \delta_{\mu\nu} \delta_{\alpha\beta} + (1.45031(13)) (\delta_{\mu\alpha} \delta_{\nu\beta} + \delta_{\nu\alpha} \delta_{\mu\beta}) \right), \quad (\text{V.25})$$

$$\frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} I_{A\nu,\mu}^{P(1c)}(0) = 0, \quad (\text{V.26})$$

$$\frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} I_{VA;\mu}^{P(2c)}(0) = \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} I_{AV;\mu}^{P(2c)}(0) = \frac{1}{2} \frac{\partial^2}{\partial(ap_\alpha)\partial(ap_\beta)} I_{V\nu,\mu}^{P(1c)}(0) \gamma_\nu \gamma_5. \quad (\text{V.27})$$

B. Tree level contribution

We consider the tree level contribution to the four quark operators given in Fig. 4. These diagrams may give a power subtraction with lower dimensional operators. We shall evaluate the amputated quark bilinear vertex function given by

$$I_{k;XY}^{(\text{sub})} = \left\langle Q_{XY}^{(k)} s_{a\alpha}(-p) \bar{d}_{b\beta}(p) \right\rangle_{\text{1PI}}. \quad (\text{V.28})$$

For each operator we have the following vertex function

$$\begin{aligned} I_{2n-1;XY}^{(\text{sub})} &= -N_c \delta_{ab} (\Gamma_X)_{\alpha\beta} \sum_{q=u,d,s} \alpha_q^{(n)} I_Y^{(2)}(m_q) \\ &\quad + \alpha_d^{(n)} \delta_{ab} I_{XY}^{(\text{sub})}(m_d)_{\alpha\beta} + \alpha_s^{(n)} \delta_{ab} I_{YX}^{(\text{sub})}(m_s)_{\alpha\beta}, \end{aligned} \quad (\text{V.29})$$

$$\begin{aligned} I_{2n;XY}^{(\text{sub})} &= -\delta_{ab} (\Gamma_X)_{\alpha\beta} \sum_{q=u,d,s} \alpha_q^{(n)} I_Y^{(2)}(m_q) \\ &\quad + N_c \alpha_d^{(n)} \delta_{ab} I_{XY}^{(\text{sub})}(m_d)_{\alpha\beta} + N_c \alpha_s^{(n)} \delta_{ab} I_{YX}^{(\text{sub})}(m_s)_{\alpha\beta}, \end{aligned} \quad (\text{V.30})$$

where we have two kinds of loop integrals in the above

$$I_{XY}^{(\text{sub})}(m) = \int \frac{d^4 l}{(2\pi)^4} \Gamma_X S_F(l, m) \Gamma_Y, \quad (\text{V.31})$$

$$I_Y^{(2)}(m) = \int \frac{d^4 l}{(2\pi)^4} \text{tr} (\Gamma_Y S_F(l, m)) = 0 \quad (\text{V.32})$$

The latter vanishes for both operator $Y = V, A$. An explicit calculation gives

$$I_{VA}^{(\text{sub})}(m) = I^{(\text{sub})}(am) \gamma_5, \quad (\text{V.33})$$

$$I_{AV}^{(\text{sub})}(m) = -I^{(\text{sub})}(am) \gamma_5, \quad (\text{V.34})$$

$$I^{(\text{sub})}(am) = \int \frac{d^4 l}{(2\pi)^4} \frac{4W(l, am)}{\bar{l}^2 + W(l, am)^2}. \quad (\text{V.35})$$

Substituting this result we obtain

$$I_{2n-1;VA}^{(\text{sub})} = -I_{2n-1;AV}^{(\text{sub})} = \alpha_d^{(n)} \delta_{ab} (\gamma_5)_{\alpha\beta} (I^{(\text{sub})}(m_d) - I^{(\text{sub})}(m_s)), \quad (\text{V.36})$$

$$I_{2n;VA}^{(\text{sub})} = -I_{2n;AV}^{(\text{sub})} = N \alpha_d^{(n)} \delta_{ab} (\gamma_5)_{\alpha\beta} (I^{(\text{sub})}(m_d) - I^{(\text{sub})}(m_s)), \quad (\text{V.37})$$

which may be evaluated with an expansion in the quark mass

$$I^{(\text{sub})}(m) = \frac{1}{a^2} m \frac{d}{d(am)} I^{(\text{sub})}(0) + \frac{1}{a} m^2 \frac{1}{2} \frac{d^2}{d(am)^2} I^{(\text{sub})}(0) + m^3 \frac{1}{6} \frac{d^3}{d(am)^3} I^{(\text{sub})}(0) + \mathcal{O}(a). \quad (\text{V.38})$$

The numerical evaluation gives

$$\frac{d}{d(am)} I^{(\text{sub})}(0) = \frac{1}{16\pi^2} (-21.46586(54)), \quad (\text{V.39})$$

$$\frac{d^2}{d(am)^2} I^{(\text{sub})}(0) = \frac{1}{16\pi^2} (-14.9157(11)). \quad (\text{V.40})$$

It may not be a good idea to expand in terms of the quark mass since these coefficients are rather large and furthermore $d^3/d(am)^3 I^{(\text{sub})}(0)$ term has an infra red divergence at $m = 0$.

This contribution introduces a mixing with the lower dimensional bilinear operator $(\bar{s} \gamma_5 d)$ multiplied with a mass difference $(m_d - m_s)$ as is given in (III.32). It is clear from (V.35) that the mixing is due to the chiral symmetry breaking effect in the Wilson fermion.

VI. RENORMALIZATION FACTOR IN $\overline{\text{MS}}$ SCHEME

We renormalize the lattice bare operators $Q_{\text{lat}}^{(k)}$ to obtain the renormalized operator $Q_{\overline{\text{MS}}}^{(k)}$. We adopt the $\overline{\text{MS}}$ scheme with DRED or NDR. The renormalization of the operator is given

by

$$Q_{\overline{\text{MS}}}^{(i)} = Z_{ij}^g Q_{\text{lat}}^{(j)} + Z_i^{\text{pen}} Q_{\text{lat}}^{\text{pen}} + Z_i^{\text{sub}} O_{\text{lat}}^{\text{sub}} \quad (\text{VI.1})$$

where $Q_{\text{lat}}^{(j)}$ is the four quark operators on the lattice, $Q_{\text{pen}}^{\text{lat}}$ is the QCD penguin operator and $O_{\text{sub}}^{\text{lat}}$ is a lower dimensional operator to be subtracted. Z_{ij}^g comes from the gluon exchanging diagrams. Z_i^{pen} is the contribution from the penguin diagrams.

A. Gluon exchanging diagrams

For gluon exchanging diagram we sum up all the contributions from three diagrams (a), (b), (c) and multiply by the color factor. Here we show its explicit form for the $\Delta S = 1$ operators.

$$I_{VA+AV}^{(k=1,3,9)} = \left(\frac{N^2 - 2}{2N} (T_V + T_A) + \frac{1}{2N} (T_S + T_P) \right) (1\tilde{\otimes}1) (V \otimes A + A \otimes V) + \frac{1}{2} (T_V + T_A - T_S - T_P) (1\tilde{\odot}1) (V \otimes A + A \otimes V) \quad (\text{VI.2})$$

$$I_{VA+AV}^{(k=2,4,10)} = \left(\frac{N^2 - 2}{2N} (T_V + T_A) + \frac{1}{2N} (T_S + T_P) \right) (1\tilde{\odot}1) (V \otimes A + A \otimes V) + \frac{1}{2} (T_V + T_A - T_S - T_P) (1\tilde{\otimes}1) (V \otimes A + A \otimes V) \quad (\text{VI.3})$$

$$I_{VA-AV}^{(k=5,7)} = \left(\frac{N^2}{2N} (T_V + T_A) - \frac{1}{2N} (T_S + T_P) \right) (1\tilde{\otimes}1) (V \otimes A - A \otimes V) + \frac{1}{2} (-T_V - T_A + T_S + T_P) (1\tilde{\odot}1) (V \otimes A - A \otimes V) \quad (\text{VI.4})$$

$$I_{VA-AV}^{(k=6,8)} = \left(\frac{N^2 - 1}{2N} (T_S + T_P) \right) (1\tilde{\odot}1) (V \otimes A - A \otimes V) \quad (\text{VI.5})$$

From these vertex functions one can easily see that the one loop correction to the four quark operators is given in a form

$$Q_{\text{one-loop}}^{(i)} = T_{ij}^{\text{lat}} Q_{\text{tree}}^{(j)}, \quad (\text{VI.6})$$

where $Q_{\text{tree}}^{(j)}$ is a tree level operator. The correction factors are given by

$$\begin{aligned} T_{11}^{\text{lat}} &= T_{22}^{\text{lat}} = T_{33}^{\text{lat}} = T_{44}^{\text{lat}} = T_{99}^{\text{lat}} = T_{10,10}^{\text{lat}} = \frac{N^2 - 2}{2N} (T_V + T_A) + \frac{1}{2N} (T_S + T_P) \\ &= \frac{g^2}{16\pi^2} \left(-\frac{N^2 + 2}{N} \ln(\lambda a)^2 + \frac{N^2 - 2}{2N} (V_V + V_A) + \frac{1}{2N} (V_S + V_P) \right), \quad (\text{VI.7}) \\ T_{55}^{\text{lat}} &= T_{77}^{\text{lat}} = \frac{N}{2} (T_V + T_A) - \frac{1}{2N} (T_S + T_P) \end{aligned}$$

$$= \frac{g^2}{16\pi^2} \left(-\frac{N^2-4}{N} \ln(\lambda a)^2 + \frac{N}{2} (V_V + V_A) - \frac{1}{2N} (+V_S + V_P) \right), \quad (\text{VI.8})$$

$$\begin{aligned} T_{66}^{\text{lat}} &= T_{88}^{\text{lat}} = \frac{N^2-1}{2N} (T_S + T_P) \\ &= \frac{g^2}{16\pi^2} \left(-4\frac{N^2-1}{N} \ln(\lambda a)^2 + \frac{N^2-1}{2N} (V_S + V_P) \right), \end{aligned} \quad (\text{VI.9})$$

$$\begin{aligned} T_{12}^{\text{lat}} &= T_{21}^{\text{lat}} = T_{34}^{\text{lat}} = T_{43}^{\text{lat}} = T_{9,10}^{\text{lat}} = T_{10,9}^{\text{lat}} = \frac{1}{2} (T_V + T_A - T_S - T_P) \\ &= \frac{g^2}{16\pi^2} \frac{1}{2} (6 \ln(\lambda a)^2 + V_V + V_A - V_S - V_P), \end{aligned} \quad (\text{VI.10})$$

$$\begin{aligned} T_{56}^{\text{lat}} &= T_{78}^{\text{lat}} = \frac{1}{2} (-T_V - T_A + T_S + T_P) \\ &= \frac{g^2}{16\pi^2} \frac{1}{2} (-6 \ln(\lambda a)^2 - V_V - V_A + V_S + V_P), \end{aligned} \quad (\text{VI.11})$$

where λ is a gluon mass introduced for infrared regularization.

The renormalization factor is given by taking a ratio of quantum correction with that in the $\overline{\text{MS}}$ scheme multiplied with the quark wave function renormalization factor Z_2

$$Z_{ii}^g(\mu a) = \frac{\left(Z_2^{\overline{\text{MS}}}\right)^2 \left(1 + T_{ii}^{\overline{\text{MS}}}\right)}{\left(Z_2^{\text{lat}}\right)^2 \left(1 + T_{ii}^{\text{lat}}\right)}, \quad (\text{VI.12})$$

$$Z_{ij}^g(\mu a) = T_{ij}^{\overline{\text{MS}}} - T_{ij} \quad (i \neq j). \quad (\text{VI.13})$$

The correction factor in the DRED $\overline{\text{MS}}$ scheme is given by

$$T_{11}^{\overline{\text{MS}}} = T_{22}^{\overline{\text{MS}}} = T_{33}^{\overline{\text{MS}}} = T_{44}^{\overline{\text{MS}}} = T_{99}^{\overline{\text{MS}}} = T_{10,10}^{\overline{\text{MS}}} = \left(\frac{N^2+2}{N}\right) V^{\overline{\text{MS}}}, \quad (\text{VI.14})$$

$$T_{12}^{\overline{\text{MS}}} = T_{21}^{\overline{\text{MS}}} = T_{34}^{\overline{\text{MS}}} = T_{43}^{\overline{\text{MS}}} = T_{9,10}^{\overline{\text{MS}}} = T_{10,9}^{\overline{\text{MS}}} = -3V^{\overline{\text{MS}}}, \quad (\text{VI.15})$$

$$T_{55}^{\overline{\text{MS}}} = T_{77}^{\overline{\text{MS}}} = \left(\frac{N^2-4}{N}\right) V^{\overline{\text{MS}}}, \quad (\text{VI.16})$$

$$T_{56}^{\overline{\text{MS}}} = T_{78}^{\overline{\text{MS}}} = 3V^{\overline{\text{MS}}}, \quad (\text{VI.17})$$

$$T_{66}^{\overline{\text{MS}}} = T_{88}^{\overline{\text{MS}}} = 4\frac{N^2-1}{N} V^{\overline{\text{MS}}}, \quad (\text{VI.18})$$

$$V^{\overline{\text{MS}}} = \frac{g^2}{16\pi^2} \left(\log\left(\frac{\mu^2}{\lambda^2}\right) + 1 \right). \quad (\text{VI.19})$$

The quark wave function renormalization factor is given by Ref. [4] and the result in the Feynman gauge is

$$\left(\frac{Z_2^{\overline{\text{MS}}}}{Z_2^{\text{lat}}}\right)(\mu a) = 1 + \frac{g^2}{16\pi^2} C_F \left(-\log(\mu a)^2 + \Sigma_1^{\overline{\text{MS}}} - \Sigma_1 \right), \quad (\text{VI.20})$$

where

$$\Sigma_1^{\overline{\text{MS}}}(\text{DRED}) = -\frac{1}{2}, \quad (\text{VI.21})$$

$$\Sigma_1 = \Sigma_1^{(0)} + c_{\text{SW}} \Sigma_1^{(1)} + c_{\text{SW}}^2 \Sigma_1^{(2)}. \quad (\text{VI.22})$$

The numerical value of $\Sigma_1^{(n)}$ is given in table II.

Substituting the above results we have

$$\begin{aligned} Z_{11}^g(\mu a) &= Z_{22}^g(\mu a) = Z_{33}^g(\mu a) = Z_{44}^g(\mu a) = Z_{99}^g(\mu a) = Z_{10,10}^g(\mu a) \\ &= 1 + \frac{g^2}{16\pi^2} \left(\frac{3}{N} \ln(\mu a)^2 + z_{11}^g \right), \end{aligned} \quad (\text{VI.23})$$

$$Z_{55}^g(\mu a) = Z_{77}^g(\mu a) = 1 + \frac{g^2}{16\pi^2} \left(-\frac{3}{N} \ln(\mu a)^2 + z_{55}^g \right), \quad (\text{VI.24})$$

$$Z_{66}^g(\mu a) = Z_{88}^g(\mu a) = 1 + \frac{g^2}{16\pi^2} \left(\frac{3(N^2 - 1)}{N} \ln(\mu a)^2 + z_{66}^g \right), \quad (\text{VI.25})$$

$$\begin{aligned} Z_{12}^g(\mu a) &= Z_{21}^g(\mu a) = Z_{34}^g(\mu a) = Z_{43}^g(\mu a) = Z_{9,10}^g(\mu a) = Z_{10,9}^g(\mu a) \\ &= \frac{g^2}{16\pi^2} (-3 \ln(\mu a)^2 + z_{12}^g), \end{aligned} \quad (\text{VI.26})$$

$$Z_{56}^g(\mu a) = Z_{78}^g(\mu a) = \frac{g^2}{16\pi^2} (3 \ln(\mu a)^2 + z_{56}^g), \quad (\text{VI.27})$$

$$Z_{65}^g(\mu a) = Z_{87}^g(\mu a) = \frac{g^2}{16\pi^2} z_{65}^g = 0. \quad (\text{VI.28})$$

$$z_{11}^g = \frac{N^2 + 2}{N} - \frac{N^2 - 2}{2N} (V_V + V_A) - \frac{1}{2N} (V_S + V_P) + 2C_F \left(\Sigma_1^{\overline{\text{MS}}} - \Sigma_1 \right), \quad (\text{VI.29})$$

$$z_{55}^g = \frac{N^2 - 4}{N} - \frac{N}{2} (V_V + V_A) + \frac{1}{2N} (V_S + V_P) + 2C_F \left(\Sigma_1^{\overline{\text{MS}}} - \Sigma_1 \right), \quad (\text{VI.30})$$

$$z_{66}^g = 4 \frac{N^2 - 1}{N} - \frac{N^2 - 1}{2N} (V_S + V_P) + 2C_F \left(\Sigma_1^{\overline{\text{MS}}} - \Sigma_1 \right), \quad (\text{VI.31})$$

$$z_{12}^g = -3 - \frac{1}{2} (V_V + V_A - V_S - V_P), \quad (\text{VI.32})$$

$$z_{56}^g = -z_{12}^g. \quad (\text{VI.33})$$

The numerical result is given in table III as an expansion in c_{SW}

$$z_{ij}^g = z_{ij}^{g(0)} + c_{\text{SW}} z_{ij}^{g(1)} + c_{\text{SW}}^2 z_{ij}^{g(2)} \quad (\text{VI.34})$$

for $N = 3$.

We need to subtract the evanescent operators $E^{(i)}$ in the $\overline{\text{MS}}$ scheme, which comes from the difference of dimensionality from four for gamma matrices in the operator vertex [7, 8].

The evanescent operators in the DRED scheme is given by

$$E^{(i)} = E_{ij}^{(\text{DRED})} E_j, \quad (\text{VI.35})$$

$$E_{1,3,9} = (1\widetilde{\otimes}1) \left(\frac{4}{n} \bar{\gamma}_\nu^L \otimes \bar{\gamma}_\nu^L - \gamma_\nu^L \otimes \gamma_\nu^L \right) \frac{2}{\epsilon}, \quad (\text{VI.36})$$

$$E_{2,4,10} = (1\widetilde{\odot}1) \left(\frac{4}{n} \bar{\gamma}_\nu^L \otimes \bar{\gamma}_\nu^L - \gamma_\nu^L \otimes \gamma_\nu^L \right) \frac{2}{\epsilon}, \quad (\text{VI.37})$$

$$E_{5,7} = (1\widetilde{\otimes}1) \left(\frac{4}{n} \bar{\gamma}_\nu^L \otimes \bar{\gamma}_\nu^R - \gamma_\nu^L \otimes \gamma_\nu^R \right) \frac{2}{\epsilon}, \quad (\text{VI.38})$$

$$E_{6,8} = (1\widetilde{\odot}1) \left(\frac{4}{n} \bar{\gamma}_\nu^L \otimes \bar{\gamma}_\nu^R - \gamma_\nu^L \otimes \gamma_\nu^R \right) \frac{2}{\epsilon}, \quad (\text{VI.39})$$

$$E_{11}^{(\text{DRED})} = E_{33}^{(\text{DRED})} = E_{99}^{(\text{DRED})} = -\frac{g^2}{16\pi^2} N, \quad (\text{VI.40})$$

$$E_{22}^{(\text{DRED})} = E_{44}^{(\text{DRED})} = E_{10,10}^{(\text{DRED})} = \frac{g^2}{16\pi^2} N, \quad (\text{VI.41})$$

$$E_{12}^{(\text{DRED})} = E_{34}^{(\text{DRED})} = E_{9,10}^{(\text{DRED})} = \frac{g^2}{16\pi^2}, \quad (\text{VI.42})$$

$$E_{21}^{(\text{DRED})} = E_{43}^{(\text{DRED})} = E_{10,9}^{(\text{DRED})} = -\frac{g^2}{16\pi^2}, \quad (\text{VI.43})$$

$$E_{55}^{(\text{DRED})} = E_{77}^{(\text{DRED})} = -\frac{g^2}{16\pi^2} 2C_F, \quad (\text{VI.44})$$

$$E_{66}^{(\text{DRED})} = E_{88}^{(\text{DRED})} = \frac{g^2}{16\pi^2} \frac{1}{N}, \quad (\text{VI.45})$$

$$E_{65}^{(\text{DRED})} = E_{87}^{(\text{DRED})} = -\frac{g^2}{16\pi^2}, \quad (\text{VI.46})$$

where n is the dimension of the loop momentum. $\bar{\gamma}_\nu^{L/R}$ and $\gamma_\nu^{L/R}$ are $n = (4 - \epsilon)$ and four dimensional gamma matrix with the chiral projection $(1 \mp \gamma_5)$.

The conversion formula to the NDR scheme is as follows [9]

$$(z_{11}^g)^{\text{NDR}} = (z_{11}^g)^{\text{DRED}} - \frac{N^2 - 6}{2N}, \quad (\text{VI.47})$$

$$(z_{55}^g)^{\text{NDR}} = (z_{55}^g)^{\text{DRED}} - \frac{N^2 - 8}{2N}, \quad (\text{VI.48})$$

$$(z_{66}^g)^{\text{NDR}} = (z_{66}^g)^{\text{DRED}} - \frac{N^2 - 4}{N}, \quad (\text{VI.49})$$

$$(z_{12}^g)^{\text{NDR}} = (z_{12}^g)^{\text{DRED}} - \frac{5}{2}, \quad (\text{VI.50})$$

$$(z_{56}^g)^{\text{NDR}} = (z_{56}^g)^{\text{DRED}} - \frac{7}{2}, \quad (\text{VI.51})$$

$$(z_{65}^g)^{\text{NDR}} = -3 \quad (\text{VI.52})$$

with corresponding evanescent operators. The numerical value of z_{ij}^g is given in table IV for NDR.

B. Penguin diagrams

Taking a summation of the finite part and the IR divergent term the one loop correction from the penguin diagram is given by

$$I_{2n-1;VA}^{\text{pen}} = \alpha_d^{(n)} T_{\text{pen}}^{\text{lat}}(p) \left(-\frac{1}{N} 1 \tilde{\otimes} 1 + 1 \tilde{\odot} 1 \right) (\gamma_\mu \gamma_5 \otimes \gamma_\mu), \quad (\text{VI.53})$$

$$I_{2n-1;AV}^{\text{pen}} = \alpha_d^{(n)} T_{\text{pen}}^{\text{lat}}(p) \left(-\frac{1}{N} 1 \tilde{\otimes} 1 + 1 \tilde{\odot} 1 \right) (\gamma_\mu \gamma_5 \otimes \gamma_\mu), \quad (\text{VI.54})$$

$$I_{2n;VA}^{\text{pen}} = 0, \quad (\text{VI.55})$$

$$I_{2n;AV}^{\text{pen}} = \left(\sum_{q=u,d,s} \alpha_q^{(n)} \right) T_{\text{pen}}^{\text{lat}}(p) \left(-\frac{1}{N} 1 \tilde{\otimes} 1 + 1 \tilde{\odot} 1 \right) (\gamma_\mu \gamma_5 \otimes \gamma_\mu), \quad (\text{VI.56})$$

$$T_{\text{pen}}^{\text{lat}}(p) = \frac{g^2}{16\pi^2} \frac{2}{3} (\ln a^2 p^2 + V_{\text{pen}}^{\text{lat}}), \quad (\text{VI.57})$$

where we adopt the on-shell condition for external quarks

$$-i\not{p}_3 + m_q = 0, \quad i\not{p}_4 + m_q = 0, \quad p = -p_3 - p_4. \quad (\text{VI.58})$$

The finite part is expanded as

$$V_{\text{pen}}^{\text{lat}} = V_{\text{pen}}^{(0)} + c_{\text{SW}} V_{\text{pen}}^{(1)} \quad (\text{VI.59})$$

with coefficients given in table V.

We notice that the above vertex corresponds to a four fermi operator of the form

$$(\bar{s}_a \gamma_\mu \gamma_5 d_a) \sum_{q=u,d,s} (\bar{q}_b \gamma_\mu q_b), \quad (\bar{s}_a \gamma_\mu \gamma_5 d_b) \sum_{q=u,d,s} (\bar{q}_b \gamma_\mu q_a) \quad (\text{VI.60})$$

and is given by a linear combination of $Q^{(3)}$, $Q^{(4)}$, $Q^{(5)}$, $Q^{(6)}$, which defines the penguin operator

$$Q^{\text{pen}} = \left(Q_{VA+AV}^{(4)} + Q_{VA-AV}^{(6)} \right) - \frac{1}{N} \left(Q_{VA+AV}^{(3)} + Q_{VA-AV}^{(5)} \right). \quad (\text{VI.61})$$

The one loop correction from the penguin diagram to the four quark operators is written as

$$Q_{\text{one-loop}}^{(i)} = (T_i^{\text{pen}})^{\text{lat}} Q_{\text{tree}}^{\text{pen}}, \quad (\text{VI.62})$$

where $Q_{\text{tree}}^{\text{pen}}$ is the penguin operator at tree level. The correction factor is given by

$$(T_i^{\text{pen}})^{\text{lat}} = \frac{g^2}{16\pi^2} \frac{C(Q^{(i)})}{3} (\ln a^2 p^2 + V_{\text{pen}}^{\text{lat}}) \quad (\text{VI.63})$$

with operator dependent factor

$$C(Q^{(1)}) = 0, \quad (\text{VI.64})$$

$$C(Q^{(2)}) = 1, \quad (\text{VI.65})$$

$$C(Q^{(3)}) = 2, \quad (\text{VI.66})$$

$$C(Q^{(4)}) = C(Q^{(6)}) = \sum_{q=u,d,s} \alpha_q^{(2)} = N_f, \quad (\text{VI.67})$$

$$C(Q^{(5)}) = C(Q^{(7)}) = 0, \quad (\text{VI.68})$$

$$C(Q^{(8)}) = C(Q^{(10)}) = \sum_{q=u,d,s} \alpha_q^{(4)} = N_u - \frac{N_d}{2}, \quad (\text{VI.69})$$

$$C(Q^{(9)}) = -1. \quad (\text{VI.70})$$

The correction factor in the $\overline{\text{MS}}$ scheme is given in a similar form

$$Q_{\text{one-loop}}^{(i)} = (T_i^{\text{pen}})^{\overline{\text{MS}}} Q_{\text{tree}}^{\text{pen}}, \quad (\text{VI.71})$$

$$(T_i^{\text{pen}})^{\overline{\text{MS}}} = \frac{g^2}{16\pi^2} \frac{C(Q^{(i)})}{3} \left(\ln \left(\frac{p^2}{\mu^2} \right) - \frac{5}{3} - c(Q^{(i)}) \right), \quad (\text{VI.72})$$

where the scheme dependent finite term is given by

$$c^{(\text{NDR})}(Q^{(2)}) = c^{(\text{NDR})}(Q^{(2n-1)}) = -1, \quad c^{(\text{NDR})}(Q^{(2n)}) = 0, \quad (\text{VI.73})$$

$$c^{(\text{DRED})}(Q^{(2)}) = c^{(\text{DRED})}(Q^{(2n-1)}) = c^{(\text{DRED})}(Q^{(2n)}) = \frac{1}{4} \quad (\text{VI.74})$$

for $n \geq 2$.

Combining these two contributions the renormalization factor for the penguin operator is given by

$$Z_i^{\text{pen}} = (T_i^{\text{pen}})^{\overline{\text{MS}}} - (T_i^{\text{pen}})^{\text{lat}} = \frac{g^2}{16\pi^2} \frac{C(Q^{(i)})}{3} (-\ln a^2 \mu^2 + z_i^{\text{pen}}), \quad (\text{VI.75})$$

$$z_i^{\text{pen}} = -V_{\text{pen}}^{\text{lat}} - \frac{5}{3} - c(Q^{(i)}). \quad (\text{VI.76})$$

Numerical value of the finite part is given in table VI.

C. Subtraction of lower dimensional operator

As was discussed in Sec. VB the lower dimensional operator

$$O_{\text{lat}}^{\text{sub}} = \bar{s} \gamma_5 d \quad (\text{VI.77})$$

mixes with the four quark operators. The subtraction factor is given by

$$Z_{2n-1}^{(\text{sub})} = -2\alpha_d^{(n)} (I^{(\text{sub})}(m_d) - I^{(\text{sub})}(m_s)), \quad (n = 3, 4), \quad (\text{VI.78})$$

$$Z_{2n}^{(\text{sub})} = -2N\alpha_d^{(n)} (I^{(\text{sub})}(m_d) - I^{(\text{sub})}(m_s)), \quad (n = 3, 4), \quad (\text{VI.79})$$

$$Z_{2n-1}^{(\text{sub})} = Z_{2n}^{(\text{sub})} = 0, \quad (n = 1, 2, 5). \quad (\text{VI.80})$$

We may be better to evaluate these factors nonperturbatively for numerical simulation.

D. Mean field improvement

The mean field improvement is given by subtracting the tadpole contribution in the renormalization factor and replacing it by a nonperturbative value u given in terms of the average plaquette $u = P^{1/4}$ for example. The tadpole contribution resides only in Σ_1 of the quark wave function renormalization factor Z_2 . The mean field improvement works for the diagonal renormalization factor Z_{ii}^g from the gluon exchanging diagram. In the improved renormalization we shall use the renormalization factor

$$u^2 Z_{ii}^{g(\text{MF})} \quad (\text{VI.81})$$

instead of Z_{ii}^g . $Z_{ii}^{g(\text{MF})}$ is given by replacing the finite term z_{ii}^g by $z_{ii}^{g(\text{MF})}$ in which the tadpole contribution is subtracted. The c_{SW} dependent part is not affected by the mean field improvement. The numerical value is given in table VII.

VII. $O(a)$ IMPROVEMENT COEFFICIENTS

In order for the on-shell $O(a)$ improvement program to work one need to adopt the rotated field for the operator

$$\psi_c = \left[1 - \frac{ar}{2} \left(z\gamma_\mu \vec{D}_\mu - (1-z)m \right) \right] \psi, \quad (\text{VII.1})$$

$$\bar{\psi}_c = \bar{\psi} \left[1 - \frac{ar}{2} \left(-z\gamma_\mu \overleftarrow{D}_\mu - (1-z)m \right) \right] \quad (\text{VII.2})$$

in addition to the improvement of the action. We shall set the on-shell condition

$$\left(\gamma_\mu \vec{D}_\mu + m_q \right) \psi_q = 0, \quad \bar{\psi}_q \left(-\gamma_\mu \overleftarrow{D}_\mu + m_q \right) = 0 \quad (\text{VII.3})$$

for the quark fields and adopt $z = 0$ for simplicity. The bare mass m_q is defined by subtracting the additive mass correction from the bare Wilson fermion mass.

A typical form of the tree level improved four fermi operator is given by

$$\mathcal{O}_{\text{lat}} = \left(1 + \frac{ar}{2} (m_1 + m_2 + m_3 + m_4)\right) (\bar{\psi}_1 \Gamma \psi_2) (\bar{\psi}_3 \Gamma' \psi_4) + \mathcal{O}(a^2), \quad (\text{VII.4})$$

which we shall adopt for our lattice bare operator. Each quark fields ψ_i has an incoming external momentum p_i . In the following we set $r = 1$.

The renormalization relation is given by

$$\begin{aligned} Q_{\text{MS}}^{(i)} = & Z_{ij}^g Q_{\text{lat}}^{(j)} + Z_i^{\text{pen}} Q_{\text{lat}}^{\text{pen}} + Z_i^{\text{sub}} O_{\text{lat}}^{\text{sub}} \\ & - g^2 a B_{ij} Q_{\text{lat}}^{(j)} - g^2 a B'_{in} O_{n,\text{lat}} - g^2 a B_q^{\text{pen}} Q_{\text{lat}}^{\text{pen}} - g^2 a C_{ij} \tilde{Q}_{\text{lat}}^{(j)} \end{aligned} \quad (\text{VII.5})$$

with $O(a)$ subtractions, where Q_{lat} is a tree level improved lattice bare operator for the $K \rightarrow \pi\pi$ decay. $O_{n,\text{lat}}$ represents four fermi operators with wrong chirality given in (VII.8) - (VII.10). B 's are proportional to the quark mass m_q . \tilde{Q} 's are dimension seven operators proportional to the quark external momentum p_μ .

A. Contribution from gluon exchanging diagrams

We shall evaluate the $O(a)$ correction for massive quarks in this subsection. The correction has already been calculated in Ref. [3] for gluon exchanging diagrams with massless quarks.

For the gluon exchanging correction all the ten operators $Q^{(i)}$ are not distinguishable but we have only four distinction $O_{VA\pm AV}^{(k)}$, where k takes even or odd for the color factor. So we shall evaluate the one loop correction to the following four operators

$$O_1 = O_{VA+AV}^{(o)}, \quad O_2 = O_{VA+AV}^{(e)}, \quad (\text{VII.6})$$

$$O_3 = O_{VA-AV}^{(o)}, \quad O_4 = O_{VA-AV}^{(e)}, \quad (\text{VII.7})$$

for which we shall need six more operators to mix with at $O(g^2 a)$

$$O_5 = O_{SP-PS}^{(o)}, \quad O_6 = O_{SP-PS}^{(e)}, \quad (\text{VII.8})$$

$$O_7 = O_{SP+PS}^{(o)}, \quad O_8 = O_{SP+PS}^{(e)}, \quad (\text{VII.9})$$

$$O_9 = O_{\tilde{T}T}^{(o)}, \quad O_{10} = O_{\tilde{T}T}^{(e)}. \quad (\text{VII.10})$$

The flavor structure shall take the form given in (III.21) - (III.25) for a practical use in $K \rightarrow \pi\pi$ decay.

We consider the following four fermi operator

$$O_n = (\Gamma_n)_{a\alpha b\beta; c\gamma d\delta} (\bar{\psi}_{1;a,\alpha} \psi_{2;b,\beta}) (\bar{\psi}_{3;c,\gamma} \psi_{4;d,\delta}), \quad (\text{VII.11})$$

$$\left(\Gamma_{VA\pm AV}^{(k)} \right)_{a\alpha b\beta; c\gamma d\delta} = (T^{(k)})_{ab;cd} (\mp V \otimes A - A \otimes V)_{\alpha\beta; \gamma\delta}, \quad (\text{VII.12})$$

$$\left(\Gamma_{SP\pm PS}^{(k)} \right)_{a\alpha b\beta; c\gamma d\delta} = (T^{(k)})_{ab;cd} (S \otimes P \pm P \otimes S)_{\alpha\beta; \gamma\delta}, \quad (\text{VII.13})$$

$$\left(\Gamma_{\tilde{T}T}^{(k)} \right)_{a\alpha b\beta; c\gamma d\delta} = (T^{(k)})_{ab;cd} (\tilde{T} \otimes T)_{\alpha\beta; \gamma\delta}, \quad (\text{VII.14})$$

where $T^{(k)}$ represents the color factor.

The one loop contribution is written as

$$I_{k;VA\pm AV}^{(a,b,c)} = J_k^{(a,b,c)} \left(1 + \frac{1}{2} a (m_1 + m_2 + m_3 + m_4) \right) I_{VA\pm AV}^{(a,b,c)} \quad (\text{VII.15})$$

including the $O(g^2 a)$ terms, where $J_k^{(a,b,c)}$ is a color factor given in Sec. IV.

As was mentioned there the one loop correction is given in terms of that to the bilinear operator (IV.21)

$$I_{VA\pm AV}^{(a+a')} = \mp \left(G_V^{(12)} \otimes A \right) + \left(V \otimes G_A^{(34)} \right) - \left(\left(G_A^{(12)} \otimes V \right) + \left(A \otimes G_V^{(34)} \right) \right), \quad (\text{VII.16})$$

$$I_{VA+AV}^{(b+b')} = \left(G_V^{(14)} \odot A + V \odot G_A^{(23)} + G_A^{(14)} \odot V + A \odot G_V^{(23)} \right), \quad (\text{VII.17})$$

$$I_{VA-AV}^{(b+b')} = -2 \left(G_S^{(14)} \odot P + S \odot G_P^{(23)} - G_P^{(14)} \odot S - P \odot G_S^{(23)} \right), \quad (\text{VII.18})$$

$$I_{VA+AV}^{(c+c')} = -2 \left(\left(G_S^{(13)} C^{-1} \otimes CP \right) + \left(SC^{-1} \otimes CG_P^{(42)} \right) - \left(G_P^{(13)} C^{-1} \otimes CS \right) - \left(PC^{-1} \otimes CG_S^{(42)} \right) \right), \quad (\text{VII.19})$$

$$I_{VA-AV}^{(c+c')} = \left(G_V^{(13)} C^{-1} \otimes CA \right) + \left(VC^{-1} \otimes CG_A^{(42)} \right) + \left(G_A^{(13)} C^{-1} \otimes CV \right) + \left(AC^{-1} \otimes CG_V^{(42)} \right), \quad (\text{VII.20})$$

where $G_\Gamma^{(ij)}$ is a one loop correction to the bilinear vertex Γ with i-th and j-th flavor contributes for the internal quark line

$$G_\Gamma^{(ij)} = \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} V_{1\mu}(p_i, l - p_i) S_F(l - p_i, m_i) \Gamma S_F(l + p_j, m_j) V_{1\nu}(-l - p_j, p_j) G_{\mu\nu}(l). \quad (\text{VII.21})$$

The quark mass and external momentum is kept non-vanishing here.

The vertex correction can be expanded in terms of the quark mass and the external momentum up to $O(a)$ according to Ref. [10]

$$G_\Gamma^{(ij)} = T_\Gamma \Gamma + T_\Gamma^{(m)} \frac{1}{2} a (m_i + m_j) \Gamma + T_\Gamma^{(p)} i a (p_i + p_j)_\mu \tilde{\Gamma}_\mu, \quad (\text{VII.22})$$

where the vertex $\tilde{\Gamma}_\mu$ for operator subtraction is given as $\tilde{\Gamma}_\mu^+$ in Ref. [10]. We notice that $O(g^2 a \log a)$ term cancels by adopting the tree level improvement condition $c_{\text{SW}} = 1$ and the on-shell condition (A.19) for the external quarks. All the coefficients are proportional to g^2 .

Performing an explicit evaluation with $c_{\text{SW}} = 1$ the $O(g^2 a)$ coefficients are given by $T_A^{(p)} = -C_A$ and $T_V^{(p)} = C_V$, where $C_{A/V}$ is defined in Ref. [10]. We notice that definition of the tensor operator $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$ is different from that in Ref. [10]. $T_P^{(p)} = T_S^{(p)} = 0$ for on-shell quarks. $T_\Gamma^{(m)}$ is equivalent to $V_\Gamma^{(1)}$ in Ref. [10] and its explicit value can be reconstructed by

$$V_\Gamma^{(1)} = -B_\Gamma + b_0 \quad (\text{VII.23})$$

in the reference. We give numerical values in tables VIII, IX for the Iwasaki gauge action in order to avoid confusion. The coefficients for the other gauge action can be reconstructed according to the example from the numerical tables in Ref. [10].

Substituting into (VII.16) - (VII.20) the one loop correction is expanded as

$$I_{VA\pm AV}^{(\text{diagram})} = I_{VA\pm AV}^{(\text{diagram})(g^2)} + I_{VA\pm AV}^{(\text{diagram})(g^2 am)} + I_{VA\pm AV}^{(\text{diagram})(g^2 ap)}. \quad (\text{VII.24})$$

Each term represents $O(g^2)$, $O(g^2 am)$ and $O(g^2 ap)$.

In this subsection we adopt the lattice scheme (IV.22) with gluon mass regulator implicitly and evaluate the full vertex correction G_1, \dots, G_4 for operators $O_{n=1\sim 4}$. For example G_1 is given by

$$G_1 = \left(1 + \frac{1}{2}a(m_1 + m_2 + m_3 + m_4)\right) \left(\Gamma_{VA+AV}^{(o)} + \sum_{\text{diagram}=a,b,c} J_o^{(\text{diagram})} I_{VA+AV}^{(\text{diagram})}\right). \quad (\text{VII.25})$$

with the tree level vertex $\Gamma_{VA+AV}^{(o)}$

According to Ref. [10] we rewrite the bare quark mass in terms of the renormalized one

$$m_q = Z_m m_{qR}, \quad (\text{VII.26})$$

$$Z_m = 1 + g^2 Z_m^{(1)}, \quad (\text{VII.27})$$

$$Z_m^{(1)} = C_F (-3L + z_m), \quad L = -\frac{1}{16\pi^2} \ln \lambda^2 a^2. \quad (\text{VII.28})$$

We then multiply the four quark vertex correction with the wave function renormalization factor $Z_{\psi_1}^{1/2} Z_{\psi_2}^{1/2} Z_{\psi_3}^{1/2} Z_{\psi_4}^{1/2}$

$$Z_{\psi_q}^{1/2} = 1 + \frac{1}{2}g^2 Z_\psi^{(1)} + \frac{1}{2}am_{qR} \left(-1 + g^2 Z_\psi^{(a)}\right), \quad (\text{VII.29})$$

$$Z_\psi^{(1)} = C_F (-L + \Sigma_1), \quad (\text{VII.30})$$

$$Z_\psi^{(a)} = C_F \left(\frac{7}{2}L + \frac{1}{2}\Sigma_1 - z_m + \Sigma_1^{(1)} \right). \quad (\text{VII.31})$$

Here Σ_1 , $\Sigma_1^{(1)}$ and z_m are constants introduced in Ref. [10] and is given in table X for $c_{\text{SW}} = 1$.

The $O(a)$ and $O(g^2 a \log a)$ terms are canceled in

$$Z_{\psi_1}^{1/2} Z_{\psi_2}^{1/2} Z_{\psi_3}^{1/2} Z_{\psi_4}^{1/2} G_i = Z_{ij}^{g(\text{lat})} \Gamma_j + g^2 a B_{ij} \Gamma_j + g^2 a C_{ij} \tilde{\Gamma}_j \quad (\text{VII.32})$$

and one can extract the renormalization factor $Z_{ij}^{g(\text{lat})}$ and the $O(a)$ coefficient B_{ij} , B'_{in} , C_{ij} . The renormalization factor $Z_{ij}^{g(\text{lat})}$ has already been used in Sec. VI to get that in the $\overline{\text{MS}}$ scheme. Although an explicit form of the $O(a)$ coefficient is given in the appendix A, we mention that the lattice bare operator $Q_{\text{lat}}^{(j)}$ is given by $O_{n=1\sim 4}$ and $O_{n,\text{lat}}$ is given by $O_{n=5\sim 10}$. The dimension seven operator $\tilde{Q}_{\text{lat}}^{(j)}$ is given by

$$\tilde{Q}^{(j)} = \left(\tilde{\Gamma}_j \right)_{a\alpha b\beta; c\gamma d\delta} \left(\bar{\psi}_{1;a,\alpha} \psi_{2;b,\beta} \right) \left(\bar{\psi}_{3;c,\gamma} \psi_{4;d,\delta} \right) \quad (\text{VII.33})$$

in terms of the vertex given in (A.72) - (A.79).

The same $O(a)$ coefficients appear in the renormalization relation (VII.5) for the $\overline{\text{MS}}$ scheme. These coefficients are written in terms of the one loop corrections $T_\Gamma^{(m)}$, $T_\Gamma^{(p)}$, $\Sigma_1 + \Sigma_1^{(1)}$ multiplied with quark masses and external momentum as will be given in the appendix.

B. Contribution from penguin diagrams

The one loop correction from the penguin diagram to the improved operator (VII.4) is given by a slight modification of the one loop vertex (V.3) and (V.4) multiplied with the tree level improvement factor

$$\left(1 + \frac{a}{2} (m_d + m_s) \right) (1 + am_q) I_{2n-1/2n; XY}. \quad (\text{VII.34})$$

However this factor shall be canceled with the tree level contribution of the wave function renormalization factor (VII.29) and we abbreviate it. We shall evaluate (V.8), (V.9) by an expansion in the quark mass and the external momentum.

Before performing the expansion we make use of the Ward-Takahashi identity (V.11), (V.12). We notice that the identity is valid on the lattice with a non-vanishing quark mass

and external momentum and impose a restriction on the one loop correction (V.10)

$$I_\mu^P(p, m_q) = (\hat{p}^2 \delta_{\mu\lambda} - \hat{p}_\mu \hat{p}_\lambda) l_\lambda^P(p, m_q) + \sigma_{\mu\lambda} \hat{p}_\lambda \tilde{l}^P(p, m_q), \quad (\text{VII.35})$$

$$\hat{p} = 2 \sin \frac{p}{2}. \quad (\text{VII.36})$$

Taking into account the Lorentz covariance each term is expanded to give the $O(a)$ contribution

$$l_\lambda^P(p, m_q) = l^P \gamma_\lambda + m_q l_m^P \gamma_\lambda + p_\lambda l_{pS}^P + p_\alpha \sigma_{\alpha\lambda} l_{pA}^P, \quad (\text{VII.37})$$

$$\tilde{l}^P(p, m_q) = \tilde{l}^P + m_q \tilde{l}_m^P + m_q^2 \tilde{l}_{mm}^P + p_\alpha \gamma_\alpha \tilde{l}_p^P + p^2 \tilde{l}_{pp}^P. \quad (\text{VII.38})$$

We substitute this expansion into a typical correction term (VII.35) and substitute further into the one loop penguin contributions (V.8) and (V.9). After a short algebraic calculation we find that only three terms contribute to the one loop penguin diagram

$$I_{V\nu,\mu}^{P(1)}(p, m_q) = -4 (\delta_{\mu\nu} p^2 - p_\mu p_\nu) \left(l^P + \tilde{l}_p^P + a m_q l_m^P \right), \quad (\text{VII.39})$$

$$I_{A\nu,\mu}^{P(1)}(p, m_q) = 0, \quad (\text{VII.40})$$

$$I_{VA;\mu}^{P(2)}(p, m_q) = I_{AV;\mu}^{P(2)}(p, m_q) = \frac{1}{2} I_{V\nu,\mu}^{P(1)}(p, m_q) \gamma_\nu \gamma_5. \quad (\text{VII.41})$$

From the clover term contribution

$$I_\mu^{P(c)}(p, m_q) = \int \frac{d^4 l}{(2\pi)^4} S_F(l - p, m_q) V_{1\mu}^{(c)}(-l + p, l) S_F(l, m_q) \quad (\text{VII.42})$$

we have the similar form of correction

$$I_{V\nu,\mu}^{P(1c)}(p, m_q) = -4 (\delta_{\mu\nu} p^2 - p_\mu p_\nu) c_{\text{SW}} \left(l^{P(c)} + \tilde{l}_p^{P(c)} + a m_q l_m^{P(c)} \right), \quad (\text{VII.43})$$

$$I_{A\nu,\mu}^{P(1)}(p, m_q) = 0, \quad (\text{VII.44})$$

$$I_{VA;\mu}^{P(2c)}(p, m_q) = I_{AV;\mu}^{P(2c)}(p, m_q) = \frac{1}{2} I_{V\nu,\mu}^{P(1c)}(p, m_q) \gamma_\nu \gamma_5. \quad (\text{VII.45})$$

The $O(g)$ terms have already been evaluated in Sec. V and we put the same result for the notation used here

$$l_1^P = \left(l^P + \tilde{l}_p^P \right) = \frac{ig}{16\pi^2} \frac{1}{3} \left(-\ln a^2 p^2 + 1.7128269(84) \right), \quad (\text{VII.46})$$

$$l_1^{P(c)} = \left(l^{P(c)} + \tilde{l}_p^{P(c)} \right) = \frac{ig}{16\pi^2} \frac{1}{3} (1.087821(3)). \quad (\text{VII.47})$$

We notice that both the $O(gam_q)$ coefficient l_m^P and $l_m^{P(c)}$ has a logarithmic IR divergence. The same regularization scheme is also used here as was adopted in Sec. V and the coefficient

is given by

$$l_m^P = \frac{ig}{16\pi^2} \frac{1}{3} \left(\frac{5}{2} \ln a^2 p^2 - 3.59121(36) \right), \quad (\text{VII.48})$$

$$l_m^{P(c)} = \frac{ig}{16\pi^2} \frac{1}{3} \left(-\frac{3}{2} \ln a^2 p^2 + 0.74846(23) \right). \quad (\text{VII.49})$$

We substitute these results into the penguin contribution (V.3) and (V.4) including all the contributions up to $O(g^2 a)$

$$\begin{aligned} I_{2n-1;VA} &= I_{2n-1;AV} \\ &= J_{\text{pen}} \sum_{q=d,s} \alpha_q^{(n)} \left(I_{VA;\mu}^{P(2)}(p, m_q) + I_{VA;\mu}^{P(2c)}(p, m_q) \right) \otimes \left(V_{1\nu}(p_3, p_4) + V_{1\nu}^{(c)}(p_3, p_4) \right) \\ &\quad \times G_{\mu\nu}(p) \\ &= 2igJ_{\text{pen}} \sum_{q=d,s} \alpha_q^{(n)} \left(\left(l_1^P + c_{\text{SW}} l_1^{P(c)} \right) \gamma_\mu \gamma_5 \otimes \gamma_\mu \right. \\ &\quad \left. + m_q \left(l_m^P + c_{\text{SW}} (l_1^P + l_m^{P(c)}) + c_{\text{SW}}^2 l_1^{P(c)} \right) \gamma_\mu \gamma_5 \otimes \gamma_\mu \right. \\ &\quad \left. + (1 - c_{\text{SW}}) \left(l_1^P + c_{\text{SW}} l_1^{P(c)} \right) \gamma_\mu \gamma_5 \otimes \frac{i}{2} (p_3 - p_4)_\mu \right), \end{aligned} \quad (\text{VII.50})$$

$$I_{2n;VA} = 0, \quad (\text{VII.51})$$

$$\begin{aligned} I_{2n;AV} &= J_{\text{pen}} \sum_{q=u,d,s} \alpha_q^{(n)} \left(I_{V\mu;\nu}^{P(1)}(p, m_q) + I_{V\mu;\nu}^{P(1c)}(p, m_q) \right) \\ &\quad \times \left(\gamma_\mu \gamma_5 \otimes V_{1\rho}(p_3, p_4) + \gamma_\mu \gamma_5 \otimes V_{1\rho}^{(c)}(p_3, p_4) \right) G_{\nu\rho}(p) \\ &= 4igJ_{\text{pen}} \sum_{q=u,d,s} \alpha_q^{(n)} \left(\left(l_1^P + c_{\text{SW}} l_1^{P(c)} \right) \gamma_\mu \gamma_5 \otimes \gamma_\mu \right. \\ &\quad \left. + m_q \left(l_m^P + c_{\text{SW}} (l_1^P + l_m^{P(c)}) + c_{\text{SW}}^2 l_1^{P(c)} \right) \gamma_\mu \gamma_5 \otimes \gamma_\mu \right. \\ &\quad \left. + (1 - c_{\text{SW}}) \left(l_1^P + c_{\text{SW}} l_1^{P(c)} \right) \left(\gamma_\mu \gamma_5 \otimes \frac{i}{2} (p_3 - p_4)_\mu \right) \right). \end{aligned} \quad (\text{VII.52})$$

Here we made use of on-shell conditions for the external momentum

$$i(\not{p}_1 + \not{p}_2) = 0, \quad i(\not{p}_3 + \not{p}_4) = 0, \quad (\text{VII.53})$$

$$(p_3 - p_4)_\mu p_\mu = 0, \quad (\text{VII.54})$$

$$i\sigma_{\mu\nu} (p_3 + p_4)_\nu = i(p_3 - p_4)_\mu - 2m_q \gamma_\mu. \quad (\text{VII.55})$$

We notice that the mixing with an operator $\gamma_\mu \gamma_5 \otimes i(p_3 - p_4)_\mu$ drops if we set the improvement coefficient $c_{\text{SW}} = 1$. The $O(g^2 a \log a)$ term in l_m^P and $l_m^{P(c)}$ cancels in a combination $l_m^P + c_{\text{SW}} (l_1^P + l_m^{P(c)})$ for $c_{\text{SW}} = 1$. The $O(g^2 a)$ improvement is accomplished just by shifting

$V_{\text{pen}}^{\text{lat}}$ in (VI.59) as

$$V_{\text{pen}}^{\text{lat}} \rightarrow V_{\text{pen}}^{\text{lat}} + m_q (0.04210(43)). \quad (\text{VII.56})$$

VIII. CONCLUSION

In this paper we have calculated the one-loop contributions to the renormalization factors for the parity odd four quark operators, which contribute to the $K \rightarrow \pi\pi$ decay amplitude, for the improved Wilson fermion action with the clover term and the Iwasaki gauge action. The operators are multiplicatively renormalizable without any mixing with wrong operators that have different chiral structures except for the lower dimensional operator. The $O(g^2a)$ improvement coefficients are also calculated for massive quarks imposing $c_{\text{SW}} = 1$ and the on-shell condition.

Acknowledgment

I would like to thank K. -I. Ishikawa, N. Ishizuka, A. Ukawa and T. Yoshié for valuable discussions. This work is supported in part by Grants-in-Aid of the Ministry of Education (Nos. 22540265, 23105701).

the one loop correction is

Appendix A: $O(a)$ contribution from gluon exchanging diagram

In this appendix we evaluate the one loop correction from the gluon exchanging diagrams with non-vanishing quark mass and the external momentum.

We start from the one loop correction (VII.16)-(VII.20) and substitute into (VII.22). The correction is expanded as (VII.24). Explicit form of the $O(g^2am)$ terms are given by

$$\begin{aligned} I_{VA\pm AV}^{(a)(g^2am)} &= \frac{1}{2} \left((m_1 + m_2) T_V^{(m)} + (m_3 + m_4) T_A^{(m)} \right) (V \otimes A) \\ &\quad \pm \frac{1}{2} \left((m_1 + m_2) T_A^{(m)} + (m_3 + m_4) T_V^{(m)} \right) (A \otimes V), \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} I_{VA+AV}^{(b)(g^2am)} &= -\frac{1}{2} \left((m_1 + m_4) T_V^{(m)} + (m_2 + m_3) T_A^{(m)} \right) (V \odot A) \\ &\quad -\frac{1}{2} \left((m_1 + m_4) T_A^{(m)} + (m_2 + m_3) T_V^{(m)} \right) (A \odot V), \end{aligned} \quad (\text{A.2})$$

$$I_{VA-AV}^{(b)(g^2 am)} = - \left((m_1 + m_4) T_S^{(m)} + (m_2 + m_3) T_P^{(m)} \right) (S \odot P) \\ + \left((m_1 + m_4) T_P^{(m)} + (m_2 + m_3) T_S^{(m)} \right) (P \odot S), \quad (\text{A.3})$$

$$I_{VA+AV}^{(c)(g^2 am)} = \left((m_1 + m_3) T_S^{(m)} + (m_2 + m_4) T_P^{(m)} \right) (SC^{-1} \otimes CP) \\ - \left((m_1 + m_3) T_P^{(m)} + (m_2 + m_4) T_S^{(m)} \right) (PC^{-1} \otimes CS), \quad (\text{A.4})$$

$$I_{VA-AV}^{(c)(g^2 am)} = \frac{1}{2} \left((m_1 + m_3) T_V^{(m)} + (m_2 + m_4) T_A^{(m)} \right) (VC^{-1} \otimes CA) \\ + \frac{1}{2} \left((m_1 + m_3) T_A^{(m)} + (m_2 + m_4) T_V^{(m)} \right) (AC^{-1} \otimes CV). \quad (\text{A.5})$$

Using the Fierz transformation with or without the charge conjugation we rearrange the spinor structure

$$(V \odot A) = -\frac{1}{2} (V \otimes A + A \otimes V) - S \otimes P + P \otimes S, \quad (\text{A.6})$$

$$(A \odot V) = -\frac{1}{2} (V \otimes A + A \otimes V) + S \otimes P - P \otimes S, \quad (\text{A.7})$$

$$(S \odot P) = \frac{1}{4} (S \otimes P + P \otimes S - V \otimes A + A \otimes V - \tilde{T} \otimes T), \quad (\text{A.8})$$

$$(P \odot S) = \frac{1}{4} (S \otimes P + P \otimes S + V \otimes A - A \otimes V - \tilde{T} \otimes T), \quad (\text{A.9})$$

$$(SC^{-1} \otimes CP) = \frac{1}{4} (S \otimes P + P \otimes S - V \otimes A - A \otimes V + \tilde{T} \otimes T), \quad (\text{A.10})$$

$$(PC^{-1} \otimes CS) = \frac{1}{4} (S \otimes P + P \otimes S + V \otimes A + A \otimes V + \tilde{T} \otimes T), \quad (\text{A.11})$$

$$(VC^{-1} \otimes CA) = -\frac{1}{2} (V \otimes A - A \otimes V) - S \otimes P + P \otimes S, \quad (\text{A.12})$$

$$(AC^{-1} \otimes CV) = -\frac{1}{2} (V \otimes A - A \otimes V) + S \otimes P - P \otimes S. \quad (\text{A.13})$$

Here we notice there appears an operator mixing with wrong chirality proportional to the quark mass difference and chiral symmetry breaking effect $(T_V - T_A)$ or $(T_S - T_P)$.

The $O(g^2 ap)$ terms are given by

$$I_{VA\pm AV}^{(a)(g^2 ap)} = T_A^{(p)} ia (p_3 + p_4)_\mu (\gamma_\mu \otimes \gamma_5) \pm T_A^{(p)} ia (p_1 + p_2)_\mu (\gamma_5 \otimes \gamma_\mu) \\ + T_V^{(p)} ia (p_1 + p_2)_\nu (\sigma_{\mu\nu} \otimes \gamma_\mu \gamma_5) \pm T_V^{(p)} ia (p_3 + p_4)_\nu (\gamma_\mu \gamma_5 \otimes \sigma_{\mu\nu}), \quad (\text{A.14})$$

$$I_{VA+AV}^{(b)(g^2 ap)} = -T_A^{(p)} ia (p_2 + p_3)_\mu (\gamma_\mu \odot \gamma_5) - T_A^{(p)} ia (p_1 + p_4)_\mu (\gamma_5 \odot \gamma_\mu) \\ - T_V^{(p)} ia (p_1 + p_4)_\nu (\sigma_{\mu\nu} \odot \gamma_\mu \gamma_5) - T_V^{(p)} ia (p_2 + p_3)_\nu (\gamma_\mu \gamma_5 \odot \sigma_{\mu\nu}), \quad (\text{A.15})$$

$$I_{VA-AV}^{(b)(g^2 ap)} = -2T_S^{(p)} ia (p_1 + p_4)_\mu (\gamma_\mu \odot \gamma_5) - 2T_P^{(p)} ia (p_2 + p_3)_\mu (1 \odot \gamma_\mu \gamma_5) \\ + 2T_P^{(p)} ia (p_1 + p_4)_\mu (\gamma_\mu \gamma_5 \odot 1) + 2T_S^{(p)} ia (p_2 + p_3)_\mu (\gamma_5 \odot \gamma_\mu), \quad (\text{A.16})$$

$$\begin{aligned}
I_{VA+AV}^{(c)(g^2ap)} &= 2T_S^{(p)}ia(p_1+p_3)_\mu (\gamma_\mu C^{-1} \otimes C\gamma_5) + 2T_P^{(p)}ia(p_2+p_4)_\mu (1C^{-1} \otimes C\gamma_\mu\gamma_5) \\
&\quad - 2T_P^{(p)}ia(p_1+p_3)_\mu (\gamma_\mu\gamma_5 C^{-1} \otimes C1) - 2T_S^{(p)}ia(p_2+p_4)_\mu (\gamma_5 C^{-1} \otimes C\gamma_\mu),
\end{aligned} \tag{A.17}$$

$$\begin{aligned}
I_{VA-AV}^{(c)(g^2ap)} &= T_A^{(p)}ia(p_2+p_4)_\mu (\gamma_\mu C^{-1} \otimes C\gamma_5) + T_A^{(p)}ia(p_1+p_3)_\mu (\gamma_5 C^{-1} \otimes C\gamma_\mu) \\
&\quad + T_V^{(p)}ia(p_1+p_3)_\nu (\sigma_{\mu\nu} C^{-1} \otimes C\gamma_\mu\gamma_5) + T_V^{(p)}ia(p_2+p_4)_\nu (\gamma_\mu\gamma_5 C^{-1} \otimes C\sigma_{\mu\nu}).
\end{aligned} \tag{A.18}$$

Using the momentum conservation relation $p_1 + p_2 + p_3 + p_4 = 0$ and the on-shell condition

$$(i\not{p}_i + m_i)\psi_i(p_i) = 0, \quad \bar{\psi}_i(p_i)(-i\not{p}_i + m_i) = 0, \tag{A.19}$$

$$(i\not{p}_i + m_i)C^{-1}\bar{\psi}_i^T(p_i) = 0, \quad \psi_i^T(p_i)C(-i\not{p}_i + m_i) = 0 \tag{A.20}$$

we rewrite the correction

$$\begin{aligned}
I_{VA\pm AV}^{(a)(g^2ap)} &= -T_A^{(p)}a(m_1 - m_2)(S \otimes P) \mp T_A^{(p)}a(m_3 - m_4)(P \otimes S) \\
&\quad + T_V^{(p)}ia(p_1 + p_2)_\nu (\sigma_{\mu\nu} \otimes \gamma_\mu\gamma_5) \pm T_V^{(p)}ia(p_3 + p_4)_\nu (\gamma_\mu\gamma_5 \otimes \sigma_{\mu\nu}),
\end{aligned} \tag{A.21}$$

$$\begin{aligned}
I_{VA+AV}^{(b)(g^2ap)} &= T_A^{(p)}a(m_1 - m_4)(S \odot P) + T_A^{(p)}a(-m_2 + m_3)(P \odot S), \\
&\quad - T_V^{(p)}ia(p_1 + p_4)_\nu (\sigma_{\mu\nu} \odot \gamma_\mu\gamma_5) - T_V^{(p)}ia(p_2 + p_3)_\nu (\gamma_\mu\gamma_5 \odot \sigma_{\mu\nu}),
\end{aligned} \tag{A.22}$$

$$\begin{aligned}
I_{VA-AV}^{(b)(g^2ap)} &= -2\left(T_S^{(p)}a(m_1 - m_4) + T_P^{(p)}a(m_2 + m_3)\right)(S \odot P) \\
&\quad + 2\left(T_P^{(p)}a(m_1 + m_4) + T_S^{(p)}a(-m_2 + m_3)\right)(P \odot S)
\end{aligned} \tag{A.23}$$

$$\begin{aligned}
I_{VA+AV}^{(c)(g^2ap)} &= 2\left(T_S^{(p)}a(m_1 - m_3) + T_P^{(p)}a(m_2 + m_4)\right)(SC^{-1} \otimes CP) \\
&\quad - 2\left(T_P^{(p)}a(m_1 + m_3) + T_S^{(p)}a(-m_2 + m_4)\right)(PC^{-1} \otimes CS),
\end{aligned} \tag{A.24}$$

$$\begin{aligned}
I_{VA-AV}^{(c)(g^2ap)} &= -T_A^{(p)}a(m_1 - m_3)(SC^{-1} \otimes CP) - T_A^{(p)}a(-m_2 + m_4)(PC^{-1} \otimes CS) \\
&\quad + T_V^{(p)}ia(p_1 + p_3)_\nu (\sigma_{\mu\nu} C^{-1} \otimes C\gamma_\mu\gamma_5) + T_V^{(p)}ia(p_2 + p_4)_\nu (\gamma_\mu\gamma_5 C^{-1} \otimes C\sigma_{\mu\nu}).
\end{aligned} \tag{A.25}$$

Taking summation of all the contributions for $VA + AV$ and $VA - AV$ and multiplying the wave function renormalization factor we extract the $O(g^2am)$ coefficients in the

$$\begin{aligned}
B_{11} &= g^2aM\left(C_F\left(\Sigma_1 + \Sigma_1^{(1)}\right) + \left(C_F - \frac{1}{2N}\right)\left(T_V^{(m)} + T_A^{(m)}\right) + \frac{1}{2N}\left(T_S^{(m)} + T_P^{(m)}\right)\right) \\
&\quad + g^2\frac{1}{N}\left(T_S^{(p)}am_{(14)} + T_P^{(p)}aM\right),
\end{aligned} \tag{A.26}$$

$$B_{12} = g^2aM\frac{1}{2}\left(T_V^{(m)} + T_A^{(m)} - T_S^{(m)} - T_P^{(m)}\right) - g^2\left(T_S^{(p)}am_{(14)} + T_P^{(p)}aM\right), \tag{A.27}$$

$$B_{13} = -g^2 am_{(12)} \left(C_F \left(T_V^{(m)} - T_A^{(m)} \right) + \frac{1}{2N} T_A^{(p)} \right), \quad (\text{A.28})$$

$$B_{14} = g^2 am_{(12)} \frac{1}{2} T_A^{(p)}, \quad (\text{A.29})$$

$$B'_{15} = g^2 am_{14} \left(-\frac{1}{2N} \left(T_V^{(m)} - T_A^{(m)} \right) + 2C_F T_A^{(p)} \right), \quad (\text{A.30})$$

$$B'_{16} = g^2 am_{14} \frac{1}{2} \left(T_V^{(m)} - T_A^{(m)} \right), \quad (\text{A.31})$$

$$B'_{17} = g^2 \left(am_{(13)} \frac{1}{2N} \left(T_S^{(m)} - T_P^{(m)} \right) + \left(2C_F + \frac{1}{2N} \right) T_A^{(p)} am_{(13)} + \frac{1}{N} \left(T_S^{(p)} am_{(12)} - T_P^{(p)} am_{(13)} \right) \right), \quad (\text{A.32})$$

$$B'_{18} = -g^2 \left(am_{(13)} \frac{1}{2} \left(T_S^{(m)} - T_P^{(m)} \right) + \frac{1}{2} T_A^{(p)} am_{(13)} + \left(T_S^{(p)} am_{(12)} - T_P^{(p)} am_{(13)} \right) \right), \quad (\text{A.33})$$

$$B'_{19} = g^2 \left(am_{(13)} \frac{1}{2N} \left(T_S^{(m)} - T_P^{(m)} \right) - \frac{1}{2N} T_A^{(p)} am_{(13)} + \frac{1}{N} \left(T_S^{(p)} am_{(12)} - T_P^{(p)} am_{(13)} \right) \right), \quad (\text{A.34})$$

$$B'_{1,10} = g^2 \left(-am_{(13)} \frac{1}{2} \left(T_S^{(m)} - T_P^{(m)} \right) + \frac{1}{2} T_A^{(p)} am_{(13)} - \left(T_S^{(p)} am_{(12)} - T_P^{(p)} am_{(13)} \right) \right), \quad (\text{A.35})$$

$$B_{21} = g^2 aM \frac{1}{2} \left(T_V^{(m)} + T_A^{(m)} - T_S^{(m)} - T_P^{(m)} \right) - g^2 \left(T_S^{(p)} am_{(14)} + T_P^{(p)} aM \right), \quad (\text{A.36})$$

$$B_{22} = g^2 aM \left(C_F \left(\Sigma_1 + \Sigma_1^{(1)} \right) + \left(C_F - \frac{1}{2N} \right) \left(T_V^{(m)} + T_A^{(m)} \right) + \frac{1}{2N} \left(T_S^{(m)} + T_P^{(m)} \right) \right) + g^2 \frac{1}{N} \left(T_S^{(p)} am_{(14)} + T_P^{(p)} aM \right), \quad (\text{A.37})$$

$$B_{23} = -\frac{1}{2} g^2 m_{(12)} \left(T_V^{(m)} - T_A^{(m)} \right), \quad (\text{A.38})$$

$$B_{24} = \frac{1}{2N} g^2 m_{(12)} \left(T_V^{(m)} - T_A^{(m)} \right) + g^2 am_{(12)} C_F T_A^{(p)}, \quad (\text{A.39})$$

$$B'_{25} = g^2 am_{(14)} T_A^{(p)}, \quad (\text{A.40})$$

$$B'_{26} = g^2 C_F m_{14} \left(T_V^{(m)} - T_A^{(m)} \right) - g^2 am_{(14)} \frac{1}{N} T_A^{(p)}, \quad (\text{A.41})$$

$$B'_{27} = -g^2 am_{(13)} \frac{1}{2} \left(T_S^{(m)} - T_P^{(m)} \right) + g^2 \left(T_A^{(p)} am_{(13)} - T_S^{(p)} am_{(12)} + T_P^{(p)} am_{(13)} \right), \quad (\text{A.42})$$

$$B'_{28} = g^2 am_{(13)} \frac{1}{2N} \left(T_S^{(m)} - T_P^{(m)} \right) + g^2 \left(-\left(C_F + \frac{1}{N} \right) T_A^{(p)} am_{(13)} + \frac{1}{N} \left(T_S^{(p)} am_{(12)} - T_P^{(p)} am_{(13)} \right) \right), \quad (\text{A.43})$$

$$B'_{29} = -g^2 am_{(13)} \frac{1}{2} \left(T_S^{(m)} - T_P^{(m)} \right) - g^2 \left(T_S^{(p)} am_{(12)} - T_P^{(p)} am_{(13)} \right), \quad (\text{A.44})$$

$$B'_{2,10} = g^2 am_{(13)} \frac{1}{2N} \left(T_S^{(m)} - T_P^{(m)} \right) + g^2 \left(C_F T_A^{(p)} am_{(13)} + \frac{1}{N} \left(T_S^{(p)} am_{(12)} - T_P^{(p)} am_{(13)} \right) \right), \quad (\text{A.45})$$

$$B_{31} = -g^2 am_{(12)} C_F \left(T_V^{(m)} - T_A^{(m)} \right) + g^2 \frac{1}{2N} T_A^{(p)} am_{(12)}, \quad (\text{A.46})$$

$$B_{32} = -g^2 \frac{1}{2} T_A^{(p)} am_{(12)}, \quad (\text{A.47})$$

$$B_{33} = g^2 aM \left(C_F \left(\Sigma_1 + \Sigma_1^{(1)} \right) + \left(C_F + \frac{1}{2N} \right) \left(T_V^{(m)} + T_A^{(m)} \right) - \frac{1}{2N} \left(T_S^{(m)} + T_P^{(m)} \right) \right) \\ - g^2 \frac{1}{N} \left(T_S^{(p)} am_{(13)} + T_P^{(p)} aM \right), \quad (\text{A.48})$$

$$B_{34} = -g^2 aM \frac{1}{2} \left(T_V^{(m)} + T_A^{(m)} - T_S^{(m)} - T_P^{(m)} \right) + g^2 \left(T_S^{(p)} am_{(13)} + T_P^{(p)} aM \right), \quad (\text{A.49})$$

$$B'_{35} = -g^2 am_{(13)} \frac{1}{2N} \left(T_V^{(m)} - T_A^{(m)} \right) - g^2 2C_F T_A^{(p)} am_{(13)}, \quad (\text{A.50})$$

$$B'_{36} = g^2 am_{(13)} \frac{1}{2} \left(T_V^{(m)} - T_A^{(m)} \right), \quad (\text{A.51})$$

$$B'_{37} = g^2 am_{(14)} \frac{1}{2N} \left(T_S^{(m)} - T_P^{(m)} \right) \\ - g^2 \left(\left(2C_F - \frac{1}{2N} \right) T_A^{(p)} am_{(14)} + \frac{1}{N} \left(T_P^{(p)} am_{(14)} - T_S^{(p)} am_{(12)} \right) \right), \quad (\text{A.52})$$

$$B'_{38} = -g^2 am_{(14)} \frac{1}{2} \left(T_S^{(m)} - T_P^{(m)} \right) + g^2 \left(-\frac{1}{2} T_A^{(p)} am_{(14)} + T_P^{(p)} am_{(14)} - T_S^{(p)} am_{(12)} \right), \quad (\text{A.53})$$

$$B'_{39} = -g^2 am_{(14)} \frac{1}{2N} \left(T_S^{(m)} - T_P^{(m)} \right) \\ + g^2 \left(\frac{1}{2N} T_A^{(p)} am_{(14)} + \frac{1}{N} \left(T_P^{(p)} am_{(14)} - T_S^{(p)} am_{(12)} \right) \right), \quad (\text{A.54})$$

$$B'_{3,10} = g^2 am_{(14)} \frac{1}{2} \left(T_S^{(m)} - T_P^{(m)} \right) - g^2 \left(\frac{1}{2} T_A^{(p)} am_{(14)} + \left(T_P^{(p)} am_{(14)} - T_S^{(p)} am_{(12)} \right) \right), \quad (\text{A.55})$$

$$B_{41} = -g^2 am_{(12)} \frac{1}{2} \left(T_V^{(m)} - T_A^{(m)} \right) - g^2 \frac{1}{2} T_A^{(p)} am_{(12)}, \quad (\text{A.56})$$

$$B_{42} = g^2 am_{(12)} \frac{1}{2N} \left(T_V^{(m)} - T_A^{(m)} \right) + g^2 \frac{1}{2N} T_A^{(p)} am_{(12)}, \quad (\text{A.57})$$

$$B_{43} = 0, \quad (\text{A.58})$$

$$B_{44} = g^2 aM C_F \left(\left(\Sigma_1 + \Sigma_1^{(1)} \right) + \left(T_S^{(m)} + T_P^{(m)} \right) \right) + g^2 2C_F \left(T_S^{(p)} am_{(13)} + T_P^{(p)} aM \right), \quad (\text{A.59})$$

$$B'_{45} = g^2 am_{(13)} \frac{1}{2} \left(T_V^{(m)} - T_A^{(m)} \right) - g^2 T_A^{(p)} am_{(13)}, \quad (\text{A.60})$$

$$B'_{46} = -g^2 am_{(13)} \frac{1}{2N} \left(T_V^{(m)} - T_A^{(m)} \right) + g^2 \frac{1}{N} T_A^{(p)} am_{(13)}, \quad (\text{A.61})$$

$$B'_{47} = -g^2 \frac{3}{2} T_A^{(p)} am_{(14)}, \quad (\text{A.62})$$

$$B'_{48} = -g^2 am_{(14)} C_F \left(T_S^{(m)} - T_P^{(m)} \right) \\ + g^2 \left(\frac{3}{2N} T_A^{(p)} am_{(14)} + 2C_F \left(T_P^{(p)} am_{(14)} - T_S^{(p)} am_{(12)} \right) \right), \quad (\text{A.63})$$

$$B'_{49} = -g^2 \frac{1}{2} T_A^{(p)} am_{(14)}, \quad (\text{A.64})$$

$$B'_{4,10} = g^2 am_{(14)} C_F \left(T_S^{(m)} - T_P^{(m)} \right) + g^2 \left(\frac{1}{2N} T_A^{(p)} am_{(14)} - 2C_F \left(T_P^{(p)} am_{(14)} - T_S^{(p)} am_{(12)} \right) \right). \quad (\text{A.65})$$

where the g^2 dependence is shown explicitly. The quark masses used here is given by

$$M = \frac{1}{4} (m_1 + m_2 + m_3 + m_4)_R, \quad (\text{A.66})$$

$$m_{(ij)} = \frac{1}{4} (m_i + m_j - m_{i'} - m_{j'})_R, \quad \{i', j'\} = \{1, 2, 3, 4\} - \{i, j\}. \quad (\text{A.67})$$

The $O(g^2 ap)$ coefficients are given as

$$C_{11} = -g^2 C_F T_V^{(p)}, \quad C_{12} = 0, \quad C_{13} = -g^2 \frac{1}{2N} T_V^{(p)}, \quad C_{14} = g^2 \frac{1}{2} T_V^{(p)}, \quad (\text{A.68})$$

$$C_{21} = -g^2 \frac{1}{2} T_V^{(p)}, \quad C_{22} = g^2 \frac{1}{2N} T_V^{(p)}, \quad C_{23} = 0, \quad C_{24} = g^2 C_F T_V^{(p)}, \quad (\text{A.69})$$

$$C_{35} = g^2 C_F T_V^{(p)}, \quad C_{36} = 0, \quad C_{37} = -g^2 \frac{1}{2N} T_V^{(p)}, \quad C_{38} = g^2 \frac{1}{2} T_V^{(p)}, \quad (\text{A.70})$$

$$C_{45} = g^2 \frac{1}{2} T_V^{(p)}, \quad C_{46} = -g^2 \frac{1}{2N} T_V^{(p)}, \quad C_{47} = g^2 \frac{1}{2} T_V^{(p)}, \quad C_{48} = -g^2 \frac{1}{2N} T_V^{(p)} \quad (\text{A.71})$$

with mixing operator vertex defined as

$$\tilde{\Gamma}_1 = 1 \otimes 1 i a (p_1 + p_2)_\nu \left((\sigma_{\mu\nu} \otimes \gamma_\mu \gamma_5) - (\gamma_\mu \gamma_5 \otimes \sigma_{\mu\nu}) \right), \quad (\text{A.72})$$

$$\tilde{\Gamma}_2 = 1 \odot 1 i a (p_1 + p_2)_\nu \left((\sigma_{\mu\nu} \otimes \gamma_\mu \gamma_5) - (\gamma_\mu \gamma_5 \otimes \sigma_{\mu\nu}) \right), \quad (\text{A.73})$$

$$\tilde{\Gamma}_3 = 1 \otimes 1 i a (p_1 + p_4)_\nu \left((\sigma_{\mu\nu} \odot \gamma_\mu \gamma_5) - (\gamma_\mu \gamma_5 \odot \sigma_{\mu\nu}) \right), \quad (\text{A.74})$$

$$\tilde{\Gamma}_4 = 1 \odot 1 i a (p_1 + p_4)_\nu \left((\sigma_{\mu\nu} \odot \gamma_\mu \gamma_5) - (\gamma_\mu \gamma_5 \odot \sigma_{\mu\nu}) \right), \quad (\text{A.75})$$

$$\tilde{\Gamma}_5 = 1 \otimes 1 i a (p_1 + p_2)_\nu \left((\sigma_{\mu\nu} \otimes \gamma_\mu \gamma_5) + (\gamma_\mu \gamma_5 \otimes \sigma_{\mu\nu}) \right), \quad (\text{A.76})$$

$$\tilde{\Gamma}_6 = 1 \odot 1 i a (p_1 + p_2)_\nu \left((\sigma_{\mu\nu} \otimes \gamma_\mu \gamma_5) + (\gamma_\mu \gamma_5 \otimes \sigma_{\mu\nu}) \right), \quad (\text{A.77})$$

$$\tilde{\Gamma}_7 = 1 \otimes 1 i a (p_1 + p_3)_\nu \left((\sigma_{\mu\nu} C^{-1} \otimes C \gamma_\mu \gamma_5) - (\gamma_\mu \gamma_5 C^{-1} \otimes C \sigma_{\mu\nu}) \right), \quad (\text{A.78})$$

$$\tilde{\Gamma}_8 = 1 \odot 1 i a (p_1 + p_3)_\nu \left((\sigma_{\mu\nu} C^{-1} \otimes C \gamma_\mu \gamma_5) - (\gamma_\mu \gamma_5 C^{-1} \otimes C \sigma_{\mu\nu}) \right). \quad (\text{A.79})$$

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TABLE I: Finite part V_Γ for bilinear operators [4]. Coefficients of the term $c_{\text{SW}}^n (n = 0, 1, 2)$ are given in the column marked as (n) . Terms proportional to c_{SW}^1 are zero for pseudoscalar P .

V			A			S			P		T		
(0)	(1)	(2)	(0)	(1)	(2)	(0)	(1)	(2)	(0)	(2)	(0)	(1)	(2)
6.275	-1.725	0.637	3.367	1.725	-0.637	2.533	6.902	-0.293	8.348	2.254	4.615	-1.150	-0.327

TABLE II: Finite constants for quark self-energy [4]. Coefficients of the term $c_{\text{SW}}^n (n = 0, 1, 2)$ are given in the column marked as (n) . Tadpole contribution is also listed.

Σ_1			
(0)	tad	(1)	(2)
4.825	7.482	-1.601	-0.973

TABLE III: Finite part z_{ij}^g of the renormalization factor from gluon exchanging diagrams. The DRED scheme is adopted. The color factor is set to $N = 3$. Coefficients of the term $c_{\text{SW}}^n (n = 0, 1, 2)$ are given in the column marked as (n) .

z_{11}^g			z_{55}^g			z_{66}^g			z_{12}^g		
(0)	(1)	(2)	(0)	(1)	(2)	(0)	(1)	(2)	(0)	(1)	(2)
-23.596	3.119	2.268	-25.183	5.420	2.922	-18.041	-4.933	-0.020	-2.381	0.451	-2.020

TABLE IV: Finite part z_{ij}^g of the renormalization factor from gluon exchanging diagrams in the NDR scheme. The color factor is set to $N = 3$. c_{SW} dependent terms are the same as that in the DRED scheme ($n = 1, 2$).

$z_{11}^{g(0)}$	$z_{55}^{g(0)}$	$z_{66}^{g(0)}$	$z_{12}^{g(0)}$	$z_{56}^{g(0)}$	$z_{65}^{g(0)}$
-24.096	-25.350	-19.708	-4.881	-1.120	-3

TABLE V: Finite part in the penguin diagram contribution on the lattice. Coefficients of the term $c_{\text{SW}}^n (n = 0, 1)$ are given in the column marked as (n) .

V_{pen}	
(0)	(1)
-1.7128	-1.0878

TABLE VI: Finite part in the renormalization factor for the penguin diagram contribution. Coefficients of the term $c_{\text{SW}}^k (k = 0, 1)$ are given in the column marked as (k) .

$z_i^{\text{pen}}(\text{DRED})^{(0)}$	$z_2^{\text{pen}}(\text{NDR})^{(0)}$	$z_{2n-1}^{\text{pen}}(\text{NDR})^{(0)}$	$z_{2n}^{\text{pen}}(\text{NDR})^{(0)}$	$(z_i^{\text{pen}})^{(1)}$
-0.2039	1.0462	1.0462	0.0461	1.0878

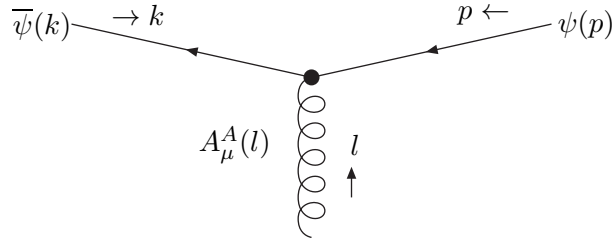


FIG. 1: One gluon interaction vertex. k and p represent incoming momentum into the vertex.

TABLE VII: Finite part $z_{ii}^{g(\text{MF})}$ of the renormalization factor from gluon exchanging diagrams for the mean field improvement given with the tadpole subtraction in the DRED and NDR scheme.

scheme	$z_{11}^{g(\text{MF})(0)}$	$z_{55}^{g(\text{MF})(0)}$	$z_{66}^{g(\text{MF})(0)}$
DRED	-3.644	-5.231	1.911
NDR	-4.144	-5.398	0.244

TABLE VIII: Coefficients of the $O(g^2 ap)$ correction to bilinear operators for the Iwasaki gauge action. The improvement coefficient is set to its tree level value $c_{\text{SW}} = 1$.

$(16\pi^2)T_A^{(p)}$	$(16\pi^2)T_V^{(p)}$	$(16\pi^2)T_P^{(p)}$	$(16\pi^2)T_S^{(p)}$
0.4519(22)	-1.1478(67)	0	0

TABLE IX: Coefficients of the $O(g^2 am)$ correction to bilinear operators for the Iwasaki gauge action. The improvement coefficient is set to its tree level value $c_{\text{SW}} = 1$.

$(16\pi^2)T_A^{(m)}$	$(16\pi^2)T_V^{(m)}$	$(16\pi^2)T_P^{(m)}$	$(16\pi^2)T_S^{(m)}$
-0.9764(47)	-0.995(13)	-1.17592(48)	-4.1201(36)

TABLE X: Coefficients of the $O(g^2)$ and $O(g^2 a)$ correction to the quark propagator for the Iwasaki gauge action. The improvement coefficient is set to its tree level value $c_{\text{SW}} = 1$.

gauge action c_1	$(16\pi^2)\Sigma_1$	$(16\pi^2)\Sigma_1^{(1)}$	$(16\pi^2)z_m$
-0.331	2.25022(44)	-9.9266(69)	-11.395(53)

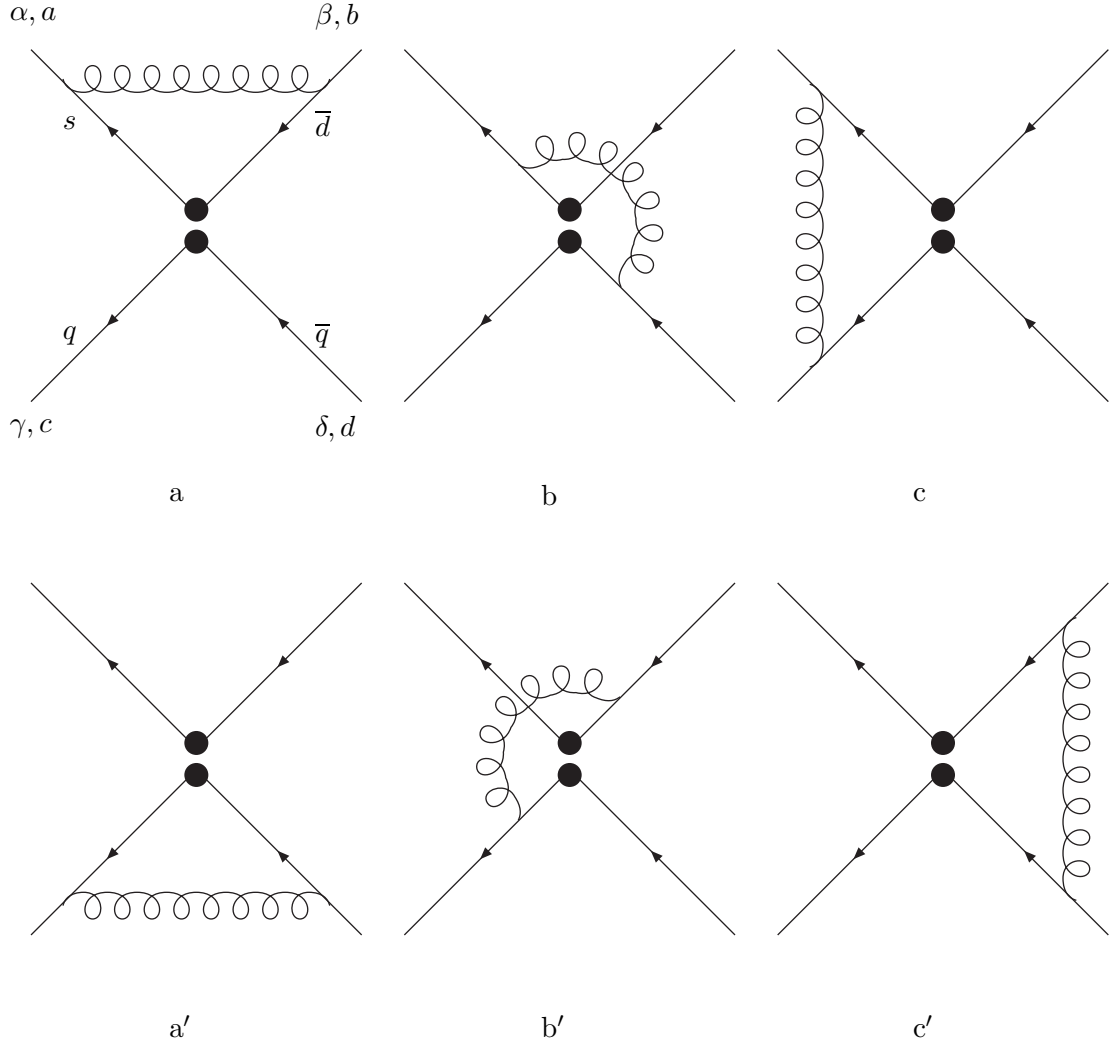


FIG. 2: One-loop vertex corrections for the four-quark operator (gluon exchanging diagram). $\alpha, \beta, \gamma, \delta$ and a, b, c, d label Dirac and color indices respectively.

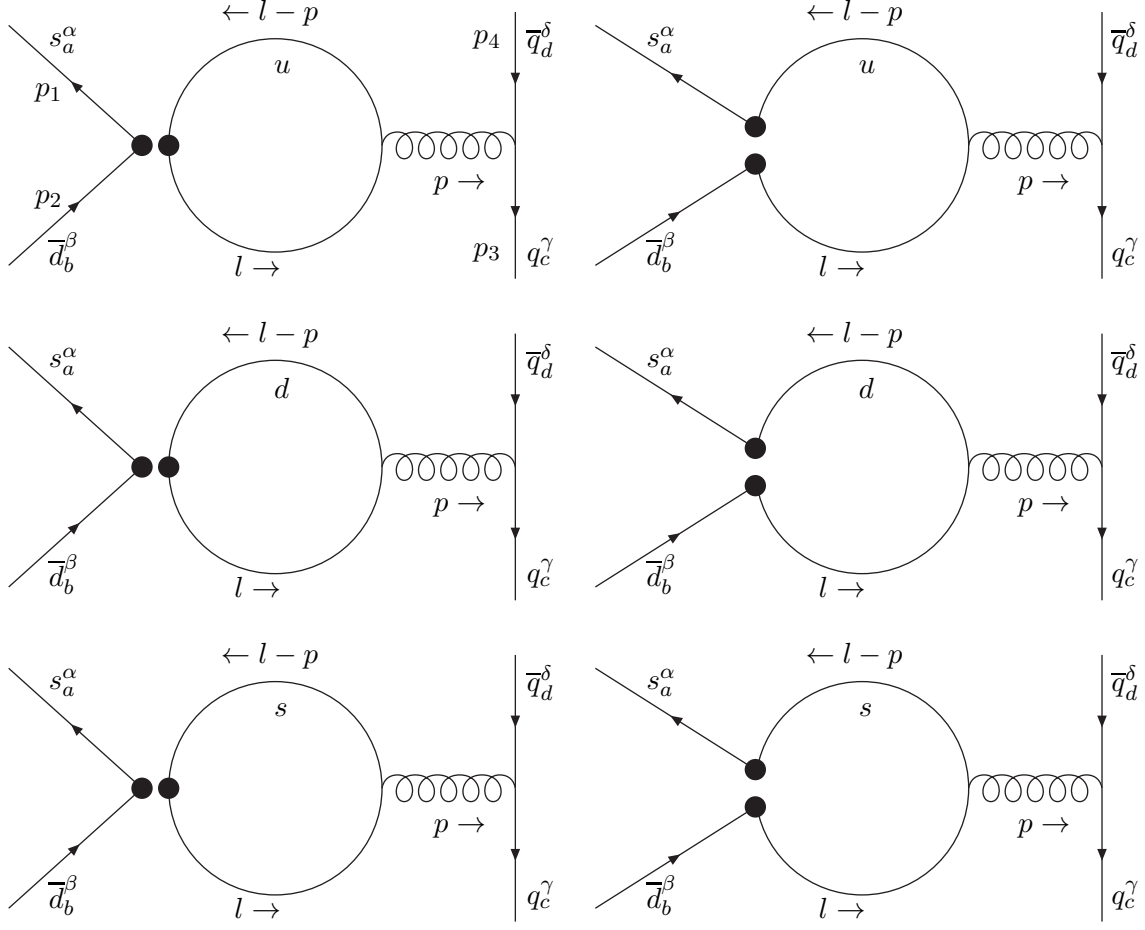


FIG. 3: One-loop vertex corrections for the four-quark operator (penguin diagram). $\alpha, \beta, \gamma, \delta$ and a, b, c, d label Dirac and color indices respectively. All external momentum p_i 's are in-coming direction.

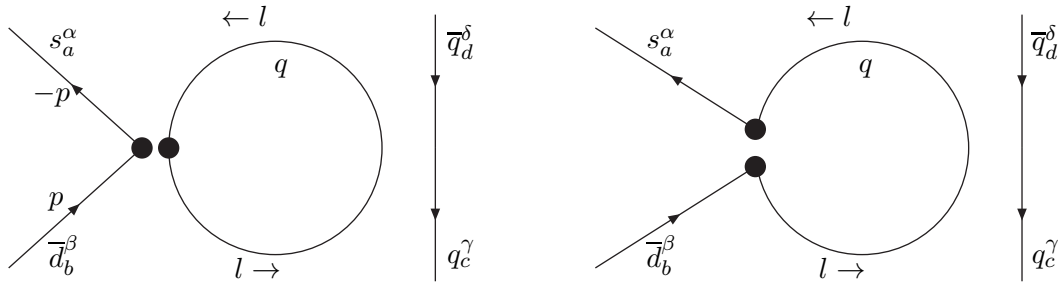


FIG. 4: Tree level contribution to the $\Delta S = 1$ four-quark operator. α, β and a, b label Dirac and color indices respectively. An external momentum p is in-coming direction